Calculus III: Final Review

Audriana Houtz

December 1, 2023

1. Suppose $\boldsymbol{u}$ and $\boldsymbol{v}$ are vectors such that $\boldsymbol{u} \cdot \boldsymbol{v} = -9$ and that $||\boldsymbol{u} \times \boldsymbol{v}|| = 9\sqrt{3}$. Find the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$.

   Solution:

2. Are the lines $\boldsymbol{r}_1 = \langle 1+t,\ 2+3t,\ 3-2t \rangle$ and $\boldsymbol{r}_2$ with $x = 1+t,\ y = 2-2t,$ and $z = 3+t$ parallel, intersecting, or skew?

   Solution:

3. Give the equation of the plane through the three points $(0,\ 1,\ 2)$, $(-1,\ 1,\ 3)$, and $(1,\ 2,\ 2)$.

Solution:

4. Are the following two parameterizations equivalent?

$$\frac{x}{1} = \frac{y-1}{1} = \frac{z+1}{2} \qquad \text{and} \qquad \boldsymbol{r}(t) = \langle t,\ 1+t,\ 1+2t \rangle$$

Solution:

5. Find $\boldsymbol{r}(t)$ that satisfies $\boldsymbol{r}(0) = \langle 2,\ 3,\ 1 \rangle$ and $\boldsymbol{r}'(t) = \langle \sin(2t),\ \cos(2t),\ 3t \rangle$.

Solution:

6. Find the normal plane of the curve $\boldsymbol{r}(t) = \langle t,\ 3\sin(t),\ 3\cos(t) \rangle$ at the point $(0,\ 0,\ 3)$.

Solution:

7. Let $f(x,\ y,\ z) = x^3y^2z + x^2y^3z^2 + xy^2z^3$. Find the partial derivative f_{zxxy}.

Solution:

8. Find the directional derivative of $f(x,\ y) = x^2y$ at the point $(1,\ 2)$ in the direction $\langle 4,\ 3\rangle$.

Solution:

9. Let $f(x,\ y) = x^2 + xy + y^2 - 4y$. Is the point $P = \left(-\frac{4}{3},\ \frac{8}{3}\right)$ a local minimum, local maximum, saddle point, or none of these?

Solution:

10. What is the maximum value of $f(x,\ y) = 5x - 3y$ subject to the constraint $x^2 + y^2 = 136$?

Solution:

11. Find an expression for the area bounded by $y = 2x$ and $y = x^2$.

Solution:

12. Compute $\int\int_D \frac{1}{\sqrt{x^2+y^2}} dA$ where D is the disk of radius 3 centered at the origin.

Solution:

13. Rewrite the iterated integral $\int_0^3 \int_0^{2-\frac{2z}{3}} \int_0^{6-2z-3y} x \; dxdydz$ as an iterated integral $dydxdz$.

Solution:

14. Set-up but do not solve the integral that computes the volume of the region below the cone $z = 2 - \sqrt{x^2 + y^2}$ and above the plane $z = 0$.

Solution:

15. Find the area of the ellipse $x^2 + 4y^2 \leq 16$ using the change of coordinates $x = 2u\cos(v)$ and $y = u\sin(v)$.

Solution:

16. Set-up an integral for the arc-length of the piece of the ellipse $x^2 + 4y^2 = 16$ from $(0,\ 2)$ to $(4,\ 0)$.

Solution:

17. Find the work done by the force field $\boldsymbol{F} = \langle 2y^3 - 6xz,\ 6xy^2 - 4y,\ 4 - 3x^2 \rangle$ if a particle of mass 1 kg moves along a curve from the point $(1,\ 0,\ -1)$ to the point $2,\ 2,\ -3)$.

Solution:

18. Compute the curl of $\boldsymbol{F} = \langle \sin(x) + yz,\ ye^z,\ e^{xy} \rangle$.

Solution:

19. Let C be the smooth simple positively-oriented curve bounding the rectangle with vertices $(0,\ 0)$, $(0,\ 1)$, $(2,\ 0)$, and $(2,\ 1)$. Compute $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$ where $\boldsymbol{F} = \langle e^y,\ xe^y + 4x \rangle$

Solution:

20. Set-up but do not solve the flux integral $\int\int_S \boldsymbol{F} \cdot d\boldsymbol{S}$ where $\boldsymbol{F} = \langle y,\ 2x,\ z \rangle$ and S is the surface $z = 5x^2 + 2y^2 - 10$ for $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$ with upwards orientation and parameterization $x = u$ and $y = v$.

Solution:

21. Let C be the curve $\boldsymbol{r}(t) = \langle \cos(t),\ \sin(t),\ 0 \rangle$. Does the hemisphere $z = -\sqrt{1 - x^2 - y^2}$ oriented downward have the wrong orientation if used in Stokes' theorem to compute a work integral around C in the direction of increasing t?

Solution:

22. Take the surface S bounded by the cylinder $x^2 + y^2 = 1$, $z = 0$, and $z = 4$ with an outward orientation. Evaluate $\int\int_S \langle x^2,\ y^2,\ xy \rangle \cdot d\boldsymbol{S}$.

Solution: