Math 10350 – Example Set 01A
Functions Review: Sections 1.1, 1.2, & 1.3

Algebra Review - Arithmetic Rules & Laws of Exponent

Complete the Arithmetic Operations below:

$a(b+c) = ab+ac$

$\frac{d}{d}\cdot\frac{a}{b}+\frac{c}{d}\cdot\frac{b}{b} = \frac{da}{db}+\frac{cb}{db} = \frac{ad+bc}{db}$

$a \times b = ab$

$a(bc) = (ab)c = abc$

$\frac{a+b}{c} = \frac{a}{c}+\frac{b}{c}$

$\frac{a}{b}\times\frac{c}{d} = \frac{ac}{bd}$

$a \div \frac{b}{c} = \frac{a}{1}\div\frac{b}{c} = \frac{a}{1}\cdot\frac{c}{b} = \frac{ac}{b}$

$\dfrac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b}\cdot\frac{d}{c} = \frac{ad}{bc}$

$(a+b)(c+d) = ac+ad+bc+bd$

$(a+b)(a-b) = a^2-ab+ba-b^2 = a^2-b^2$

$(a+b)^2 = (a+b)(a+b) = a^2+ab+ba+b^2 = a^2+2ab+b^2$

$(a-b)^2 = (a-b)(a-b) = a^2-ab-ba+b^2 = a^2-2ab+b^2$

Complete the Laws of Exponents below:

$a^m \cdot a^n = a^{m+n}$

$(ab)^m = a^m b^m$

$\frac{a^m}{a^n} = a^{m-n}$; $a \neq 0$

$a^0 = 1$; $a \neq 0$

$a^{1/m} = \sqrt[m]{a}$

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$; $b \neq 0$

$\frac{1}{b^m} = b^{-m}$

$(a^m)^n = a^{mn}$

1. Give the lowest common denominator of fractions or rational functions in the sums below then evaluate the sum giving your answer as a single rational number or function with no common factors between its numerator and denominator.

$$\frac{3}{x^2} - \frac{x}{x^2-4} - \frac{2}{x^2+2x}$$

lowest common denominator: smallest multiple shared between the denominators of the given fractions

Step 1: factor & reduce

$\frac{3}{x^2} - \frac{x}{(x+2)(x-2)} - \frac{2}{x(x+2)}$

note that there are no coefficients to find LCM of first

Step 2: write all factors without powers

$x, x+2, x-2$

Step 3: largest power from denom.

$x^2, (x+2)^1, (x-2)^1$

Step 4: product of list is LCD

$x^2(x+2)(x-2)$

Step 5: multiply by what's missing

$$\frac{3(x+2)(x-2) - x(x^2) - 2(x-2)}{x^2(x+2)(x-2)}$$

$$= \frac{3(x^2-4) - x^3 - 2x + 4}{x^2(x+2)(x-2)} = \frac{3x^2 - 12 - x^3 - 2x + 4}{x^2(x+2)(x-2)} = \frac{-x^3+3x^2-2x-8}{x^2(x+2)(x-2)}$$

numerical example: $\frac{5}{8}$ and $\frac{11}{12}$

- multiples of 8: 8, 16, 24, 32, 40, 48, ...
- multiples of 12: 12, 24, 36, 48, ...
- common multiples: 24, 48, ...
- least common multiple: 24

adding fractions: $\frac{5}{8} + \frac{11}{12}$

- common denominator: 24
- new fractions: $\frac{5}{8} = \frac{5}{8} \cdot \frac{3}{3} = \frac{15}{24}$ & $\frac{11}{12} = \frac{11}{12} \cdot \frac{2}{2} = \frac{22}{24}$
- add numerators: $\frac{15}{24} + \frac{22}{24} = \frac{37}{24}$

2. Simplify the following expression giving your answer in the form $\frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ has no common factors.

$$\frac{(x^2+2)^3 \cdot 4 - (4x+1) \cdot 3(x^2+2)^2 \cdot 2x}{(x^2+2)^6}$$

reduced to lowest terms: all common factors from the numerator and denominator have been canceled

Step 1: identify common factors

(x^2+2)

Step 2: determine lowest power

$(x^2+2)^2$

Step 3: remove factor from all terms

$$\frac{(x+2) \cdot 4 - (4x+1) \cdot 3 \cdot 2x}{(x^2+2)^4}$$

$$= \frac{4x+8 - 6x(4x+1)}{(x^2+2)^4}$$

$$= \frac{4x+8-24x^2-6x}{(x^2+2)^4}$$

$$= \frac{-24x^2-2x+8}{(x^2+2)^4}$$

numerical example: $\frac{12}{8}$

- factor completely: $\frac{12}{8} = \frac{(2)(2)(3)}{(2)(2)(2)}$
- cancel matching terms: $\frac{(2)(2)(3)}{(2)(2)(2)} = \frac{3}{2}$

common mistake: $\frac{x-1}{x}$

- you can not cancel x: $\frac{x-1}{x} \neq \frac{-1}{1}$
- plug in $x=4$ to see why: $\frac{4-1}{4} = \frac{3}{4} \neq \frac{4-1}{4} = -1$

Alternative: multiply by $\frac{(\text{factor})^{\text{-power}}}{(\text{factor})^{\text{-power}}}$

$$\frac{(x^2+2)^{-2}}{(x^2+2)^{-2}} \left(\frac{(x^2+2)^3 \cdot 4 - (4x+1) \cdot 3(x^2+2)^2 \cdot 2x}{(x^2+2)^6} \right)$$

$$= \frac{(x^2+2)^{3-2} \cdot 4 - (4x+1) \cdot 3(x^2+2)^{2-2} \cdot 2x}{(x^2+2)^{6-2}}$$

$$= \frac{(x^2+2) \cdot 4 - (4x+1) \cdot 3 \cdot 2x}{(x^2+2)^4}$$

Definition A **function** is a rule that assigns a (single) value y (in the range) to each value x (in the domain).

(Basic Functions) Give an example for each type of basic functions below and give their general form:

A. Power Function:

An example: $4x^3$

General form: ax^n

B. Polynomial Function:

An example: $3x^2 + 5x - 2$

General form: $a_n x^n + \ldots + a_0 x^0$

Special Cases

Linear functions: $ax + b$

Quadratic functions: $ax^2 + bx + c$

C. Rational Function:

An example: $\frac{5x+1}{3x^2-9}$

General form: $\frac{a_n x^n + \ldots + a_0 x^0}{b_n x^n + \ldots + b_0 x^0}$

D. Exponential Function:

An example: $2e^{3x^2}$

General form: ae^{bx^n}

E. Logarithmic Function:

An example: $7\log_{10}(15x)$

General form: $a\log_b(cx)$

F. Trigonometric Function:

$a\sin(bx)$
$a\cos(bx)$
$a\tan(bx)$