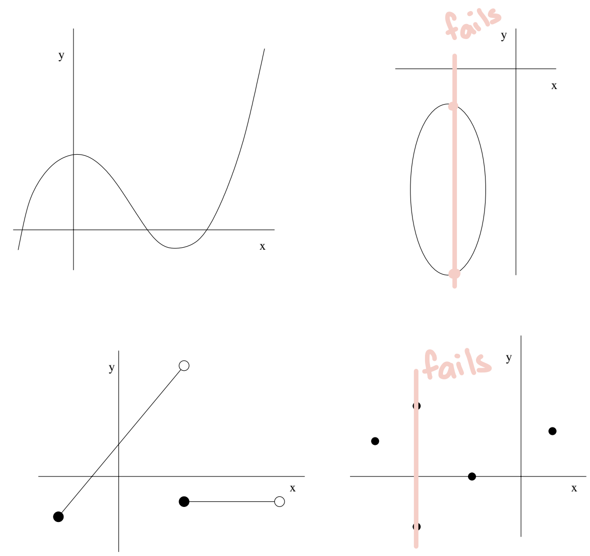
Math 10350 – Example Set 01B
Functions Review: Sections 1.1, 1.2, & 1.3

1. The quantity y relates to x in each of the following graphs. For each graph determine whether y is a function of x.



function: a relation from a set of inputs (domain) to a set of outputs (range) where each input is related to exactly one output

vertical line test: a visual way to determine if a curve is a graph of a function or not

Price, Revenue, Cost & Profit. Write an equation that connects the revenue from the sale of a certain product, the number of the product sold (or demand), and selling price of one unit of the product. How does revenue differ from the profit from the sale of the product?

Revenue: how much money people gave you
Profit: remove how much you spent $C(x)$

$R(x) =$ sale price $\cdot$ sold $= s \cdot x$
$P(x) =$ revenue $-$ cost $= R(x) - C(x)$

2. (An application of Functions) A electronic company decides to set the sale price of a sound card at \$60 a piece for a monthly demand of 100 pieces. The sale price drops to \$50 a piece for a monthly demand of 200 pieces.

2a. Assuming that the sale price for one sound card is a linear function of the size of the monthly demand, find a formula for the sale price s dollars per sound card in terms of the size x of the monthly demand. What is the revenue function from the sales of the sound card? $\left(-\frac{x}{10} + 70; -\frac{x^2}{10} + 70x\right)$

2b. Suppose further that the company has a monthly overhead cost of \$5000 for producing the sound cards and a cost of \$10 for producing each piece of the sound card. What is the monthly profit from the sales of the sound card in terms of month production assuming that all items produced are sold? $\left(-\frac{x^2}{10} + 60x - 5000\right)$

2a. Step 1: Set up function for sale price
- we have two points: $(100, 60)$, $(200, 50)$
- we need point-slope or point-intercept
- find slope using two points: $m = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{50-60}{200-100} = \frac{-10}{100} = \frac{-1}{10}$

- point-slope equation: $y - y_0 = m(x - x_0)$

$y - 60 = -\frac{1}{10}(x - 100)$
$y - 60 = -\frac{1}{10}x + 10$
$y = -\frac{1}{10}x + 70$
$s(x) = -\frac{1}{10}x + 70$

Step 2: Set up revenue function
- revenue function: $R(x) = s(x) \cdot x$

$R(x) = (-\frac{1}{10}x + 70) \cdot x$
$R(x) = -\frac{1}{10}x^2 + 70x$

2b. Step 1: Set up cost function
- cost function: $C(x) = (\text{cost per item})x + (\text{fixed cost})$

$C(x) = 10 \cdot x + 5000$

Step 2: Set up profit function
- profit function: $P(x) = R(x) - C(x)$

$P(x) = -\frac{1}{10}x^2 + 70x - (10x + 5000)$
$P(x) = -\frac{1}{10}x^2 + 70x - 10x - 5000$
$P(x) = -\frac{1}{10}x^2 + 60x - 5000$

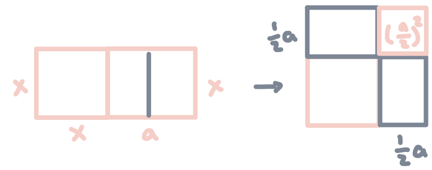
Completing the Square Notes

Completing the square is an algebraic process applied to quadratic expressions of the form $x^2 + ax$ to obtain a perfect square. Specifically we want to find a positive number b such that

$$x^2 \quad + \quad ax \quad + \quad \underline{\quad b \quad} \quad = \quad \underline{\quad (x+c)^2 \quad}$$

where both b and c are to be determined.

Geometric interpretation of completing the square.



Interpret x^2 as the area of a square and

Interpret ax as the area of rectangle with side lengths a & x

Piece together the square and rectangles together to see the method of completing the square.

Examples. Fill in the blanks for each quadratic expressions of the form $x^2 + ax$ below to obtain a perfect square.

(a) $x^2 \quad + \quad 6x \quad + \quad \underline{\quad 9 \quad} \quad = \quad \underline{\quad (x+3)^2 \quad}$ $\qquad a=6: \ (\frac{1}{2}a)^2 = (3)^2 = 9$

(b) $x^2 \quad - \quad 4x \quad + \quad \underline{\quad 4 \quad} \quad = \quad \underline{\quad (x-2)^2 \quad}$ $\qquad a=-4: \ (\frac{1}{2}a)^2 = (-2)^2 = 4$

(c) $x^2 \quad + \quad 5x \quad + \quad \underline{\quad \frac{25}{4} \quad} \quad = \quad \underline{\quad (x+\frac{5}{2})^2 \quad}$ $\qquad a=5: \ (\frac{1}{2}a)^2 = (\frac{5}{2})^2 = \frac{25}{4}$

3. (Completing the Square Practice) Rewrite each of the following quadratic functions in the form $A(x + B)^2 + C$: **(i)** $x^2 - 6x - 5$, **(ii)** $-2x^2 - 8x + 1$. **(iii)** Graph the quadratic function in (ii). State the coordinates of the vertex and the **equation** of the axis of symmetry.

4. Rewrite the monthly profit function in Q2 in the form $A(x + B)^2 + C$.

a. By scaling and translating x^2, graph the monthly profit function labelling the axis of symmetry, vertical intercept and vertex.

b. What is the maximum profit the company can make and when does that happen?

3. (Completing the Square Practice) Rewrite each of the following quadratic functions in the form $A(x+B)^2+C$: **(i)** x^2-6x-5, **(ii)** $-2x^2-8x+1$. **(iii)** Graph the quadratic function in (ii). State the coordinates of the vertex and the **equation** of the axis of symmetry.

3(i) x^2-6x-5

- find $B=\frac{b}{2}$, $b=-6$
  $B=-3$
- complete the square
  $x^2-6x-5+9-9$
  $(x^2-6x+9)-5-9$
  $(x-3)^2-14$

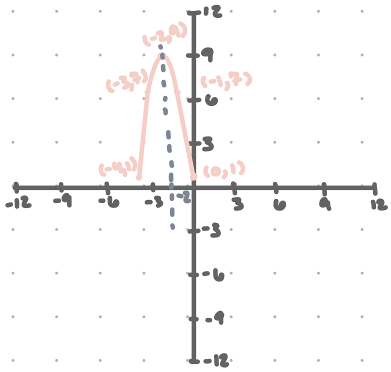
In order to complete the square we must add and subtract the missing piece $(\frac{b}{2})^2$

3(ii) $-2x^2-8x+1$

- factor: $-2(x^2+4x)+1$
- find B: $B=\frac{b}{2}$; $b=4$
  $B=2$
- complete the square
  $-2x^2-8x+1+8-8$
  $-2(x^2+4x+4)+1+8$
  $-2(x+2)^2+9$

3(iii) Graph $-2(x+2)^2+9$

- parent function: $y=a(x-h)^2+k$
  ↳ (h,k) is the vertex
  ↳ a is reflection & multiplier
  ↳ axis of symmetry: $x=h$
- for our function
  ↳ $(h,k)=(-2,9)$
  ↳ $a=-2$ i.e. flip & stretch
  ↳ axis of symmetry: $x=-2$



The addition of a coefficient on x^2 changes the missing piece to $a(\frac{b}{2})^2$

4. Rewrite the monthly profit function in Q2 in the form $A(x+B)^2+C$.

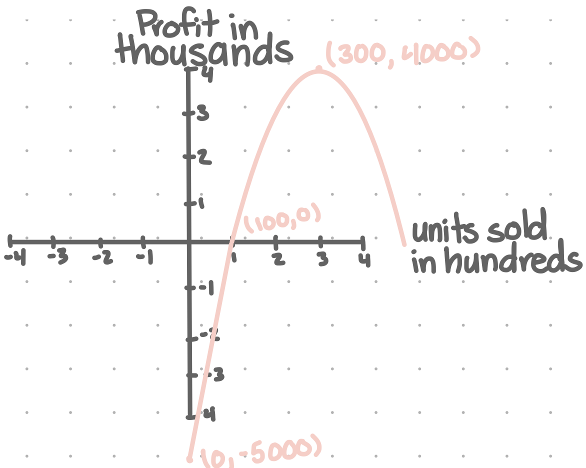
$P(x)=-\frac{1}{10}x^2+60x-5000$
$=-\frac{1}{10}(x^2-600x)-5000$

- complete the square: $a(\frac{b}{2})^2-a(\frac{b}{2})^2$
  $-\frac{1}{10}(x^2-600x)-5000+\frac{1}{10}(300)^2-\frac{1}{10}(300)^2$
  $=-\frac{1}{10}(x-300)^2-5000+9000$
  $=-\frac{1}{10}(x-300)^2+4000$

a. By scaling and translating x^2, graph the monthly profit function labelling the axis of symmetry, vertical intercept and vertex.

Graph $P(x)=-\frac{1}{10}(x-300)^2+4000$

- parent function: $y=a(x-h)^2+k$
  ↳ (h,k) is the vertex
  ↳ a is reflection & multiplier
  ↳ axis of symmetry: $x=h$
- for our function
  ↳ $(h,k)=(300,4000)$
  ↳ $a=-\frac{1}{10}$ i.e. flip & compress
  ↳ axis of symmetry: $h=300$ units sold



b. What is the maximum profit the company can make and when does that happen?

The maximum profit P(x) is at the vertex (300,4000). The company makes a maximum of $4000 when they sale 300 units.