Math 10350 – Example Set 02A
Quadratic Functions: Section 1.6
Basic Exponential Equations: Section 1.6

1. (Completing the Square Review) A particle moving in a straight line has position in meters, measured from a fixed point **O** on the straight line, at time t seconds is given by

$$s(t) = 5 - 4t + 3t^2$$

(i) Sketch the graph of $s(t)$. (ii) Find the time at which the particle is **closest** to the point **O**. (iii) How far can the particle be from the point **O**?

2. (Sect 1.6) Solve the following equations: (a) $4^x = \frac{1}{8}$; (b) $3 \cdot 9^{x+1} = 81^x$.

1. s(t) is the distance from the point O, there is no gaurantee that the particle goes through O i.e. s(t) may never be 0.

(i) Sketch

$s(t) = A(x-h)^2 + k$

$s(t) = 3t^2 - 4t + 5$

$s(t) = 3(t^2 - \frac{4}{3}t) + 5$

complete the square

$b = -\frac{4}{3}$; $(\frac{1}{2}b)^2 = (-\frac{4}{6})^2 = (-\frac{2}{3})^2 = \frac{4}{9}$

add & subtract inside parenthesis

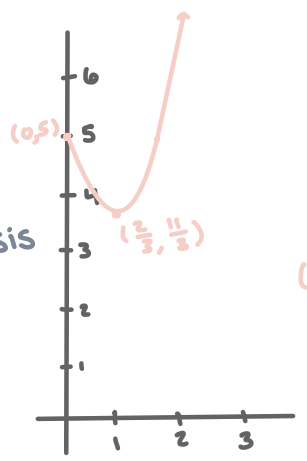
$s(t) = 3(t^2 - \frac{4}{3}t + \frac{4}{9} - \frac{4}{9}) + 5$

$= 3((t-\frac{2}{3})^2 - \frac{4}{9}) + 5$

$= 3(t-\frac{2}{3})^2 - \frac{4}{3} + 5$

$s(t) = 3(t-\frac{2}{3})^2 + \frac{11}{3}$

$(h,k) = (\frac{2}{3}, \frac{11}{3})$



(ii) minimum distance

s(t) = distance

minimum = vertex

$\frac{11}{3}$ meters away from O at time $\frac{2}{3}$ seconds

(iii) maximum distance

there is no maximum

note: parabolas have a min or a max not both

2. Solve the following equations:

(a) $4^x = \frac{1}{8}$

Step1: same base

$(2^2)^x = (2)^{-3}$

Step2: cancel base

$2^{2x} = 2^{-3}$

Step3: solve algebra

$2x = -3$

$x = -\frac{3}{2}$

eventually:

$\log_4 4^x = \log_4 \frac{1}{8}$

$x \log_4 4 = \log_4 2^{-3}$

$x = -3\log_4 2$

Note: $\log_4 2 = \frac{1}{2}$

$x = -\frac{3}{2}$

(b) $3 \cdot 9^{x+1} = 81^x$

$3 \cdot (3^2)^{x+1} = (3^4)^x$

$3 \cdot 3^{2x+2} = 3^{4x}$

$3^{2x+3} = 3^{4x}$

$2x+3 = 4x$

$3 = 2x$

$\frac{3}{2} = x$

eventually:

$\log_9(3 \cdot 9^{x+1}) = \log_9(9^{2x})$

$\log_9(3) + \log_9(9^{x+1}) = \log_9(9^{2x})$

$\frac{1}{2} + x + 1 = 2x$

$\frac{3}{2} = x$