1. Solve for x for each of the following equations: (a) $2\ln(x-3) = 3$, (b) $\log_2(2x+3) = \log_2(x+1) + 2$.

(a) $2\ln(x-3)=3$

$\ln(x-3) = \frac{3}{2}$

$\cancel{e}^{\ln(x-3)} = e^{3/2}$

$x-3 = e^{3/2}$

$x = e^{3/2} - 3$

we need to isolate x to solve

$e^{\ln(x)} = x$

(b) $\log_2(2x+3) = \log_2(x+1)+2$

$2^{\log_2(2x+3)} = 2^{\log_2(x+1)+2}$

$\cancel{2^{\log_2}}(2x+3) = \cancel{2^{\log_2}}(x+1) \cdot 2^2$

$2x+3 = 2^2(x+1)$

$2x+3 = 4(x+1)$

$2x+3 = 4x+4$

$-1 = 2x$

$-\frac{1}{2} = x$

inverse $\log_2 x$ is 2^x

must raise EVERYTHING

$x^{a+b} = x^a x^b$

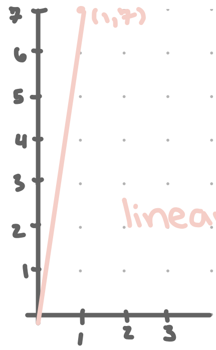
$a^{\log_a(x)} = x$

2a. It is said that one dog year equals seven human years. Chart a graph of dog years D verses human years H. What kind of function is H in terms of D?

(a) $\frac{1}{7} = \frac{D}{H}$ ratio for 1 dog year per 7 human years

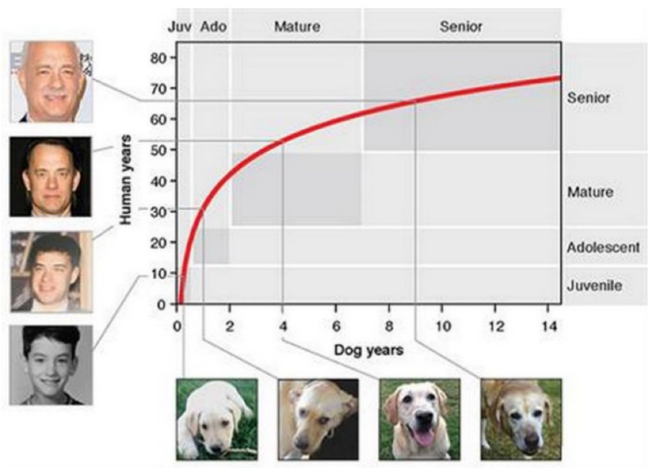
$H = 7D$

Dog	1	2	3	4	5
Human	7	14	21	28	35



linear function

2b. A new genomics research that studies aging in mammals (**) says that the equation between dog years and humans years is much more complicated: $H = 16\ln(D) + 31$. (i) What is the human years of a dog when it is one year old? (ii) What is the dog age of a 21 year old human? (**): Quantitative Translation of Dog-to-Human Aging by Conserved Remodeling of the DNA Methylome



(i) Given D=1. Find H.

$H = 16\ln(1)+31$

$= 16\cdot(0)+31$

$= 31$

1 dog year = 31 human years

(ii) Given H=21. Find D.

$21 = 16\ln(D)+31$

$-10 = 16\ln(D)$

$-\frac{10}{16} = \ln(D)$

$-\frac{5}{8} = \ln(D)$

$e^{-5/8} = \cancel{e}^{\ln(D)}$

$e^{-5/8} = D$

21 human years ≈ 0.535 dog years

Doubling time and Half life.

The **doubling time** of a quantity growing exponentially as time progress is the amount of time needed for

the initial amount to double

The **half life** of a quantity decaying exponentially as time progress is the amount of time needed for

the initial amount to be cut in half

3. Recent experiments on viability of the coronavirus indicates that it reduces exponentially on various surfaces. The half life of the coronavirus on glass is estimated to be about 14 hours. (a) Starting with 100% initially, find a formula in the form $A \cdot b^t$ for the percentage of the virus on glass after t hours. (b) If we consider the virus no longer infectious (or viable) after it is reduced to 1% or less, estimate how long will the virus remain infectious on glass.

$(100(1/2)^{t/14})$

Step 1: Set up $A(t) = A_0 \cdot b^t$

half life = 14 hours => (14, 50)

initial value = 100% => (0, 100)

=> $A_0 = 100$

Find b using (14,50)

$50 = A(14) = 100b^{14}$

$\frac{1}{2} = b^{14}$

$(\frac{1}{2})^{1/14} = b$

$A(t) = 100(\frac{1}{2}^{1/14})^t$

$= 100(\frac{1}{2})^{t/14}$

$= 100 \cdot (2)^{-t/14}$

Step 2: Find viability

not viable after time t s.t. A(s) = 1

$1 = 100(2)^{-t/14}$

$\frac{1}{100} = (2)^{-t/14}$

$\log_2(\frac{1}{100}) = \cancel{\log_2}(2)^{-t/14}$

$\log_2(\frac{1}{100}) = -t/14$

$-4\log_2(100) = t$

$-26.575 = t$

$t \approx 93.01$

Discrete Compound Interest. If P dollars (principal) is deposited in an account at annual interest rate r (in decimal form) compounded n times a year, then the balance of the account A after t years is given by

$$\boxed{A(t) = P\left(1 + \frac{r}{n}\right)^{nt}}$$

4. Find a formula for the balance A for an investment with principle \$1000 earning interest at a rate of 5% compounded (a) daily, (b) weekly, (c) monthly and (d) quarterly. Compare the balances when the interest are compounded daily, weekly and quarterly after 10 years. What could you say?

(a) P=1000, r=0.05, n=365

$A(t) = 1000(1 + \frac{0.05}{365})^{365t}$

$A(10) = 1000(1 + \frac{0.05}{365})^{365(10)}$

$= 1648.66$

(b) P=1000, r=0.05, n=52

$A(t) = 1000(1 + \frac{0.05}{52})^{52t}$

$A(10) = 1000(1 + \frac{0.05}{52})^{52 \cdot 10}$

$= 1648.33$

(c) P=1000, r=0.05, n=12

$A(t) = 1000(1 + \frac{0.05}{12})^{12t}$

$A(10) = 1000(1 + \frac{0.05}{12})^{12 \cdot 10}$

$= 1647.01$

(c) P=1000, r=0.05, n=4

$A(t) = 1000(1 + \frac{0.05}{4})^{4t}$

$A(10) = 1000(1 + \frac{0.05}{12})^{4 \cdot 10}$

Continuous Compound Interest. If P dollars is deposited in an account at annual interest rate r (in decimal form) compounded **continuously**, then the balance of the account A after t years is given by

$$\boxed{A(t) = Pe^{rt}}$$

5. Write down a formula for the balance A for an investment with principle \$1000 earning interest at a rate of 5% compounded continuously after t years. Find the doubling time of the account in years and months (to the nearest integer)? Does the doubling time depends on the principal? ($1000e^{0.05t}$)

(a) $P = 1000,\ r = 0.05$
$A(t) = 1000e^{0.05t}$

(b) doubling time: $1000e^{0.05t} = 2000$
$1000e^{0.05t} = 2000$
$e^{0.05t} = 2$
$\ln(e^{0.05t}) = \ln(2)$
$0.05t = \ln(2)$
$t = \frac{\ln(2)}{0.05}$
≈ 13.86 OR 13 years 10 months

(c) Principle & doubling time
Doubling time is the amount of time it takes to double the initial investment i.e.
$2P = Pe^{rt}$
$2 = e^{rt}$
the principal cancels out proving that it does not change doubling time

6. You like to see your investment triple every 20 years. At what rate should your investment be growing per annum if interest is (a) compounded quarterly and (b) continuous?

(a) $4(3^{1/80} - 1) \approx 0.0553$ ie 5.53% , (b) $\frac{\ln 3}{20} \approx 0.0549$ ie 5.49%

Goal: triple when $t=20$

(a) compound quarterly
$3P = P(1+\frac{r}{4})^{4\cdot 20}$
$3 = (1+\frac{r}{4})^{80}$
$3^{1/80} = 1+\frac{r}{4}$
$3^{1/80} - 1 = \frac{r}{4}$
$4(3^{1/80} - 1) = r$
$0.0553 = r$
5.53%

(b) compound continuously
$3P = Pe^{r\cdot 20}$
$3 = e^{20r}$
$\ln(3) = \cancel{\ln}(e^{20r})$
$\ln(3) = 20r$
$\frac{\ln(3)}{20} = r$
$0.0549 = r$
5.49%