Math 10350 – Example Set 04B

1. Find the equation(s) of the tangent line(s) to the graph of $y = x^3 + 2$ is parallel to the line $24x - 2y = 3$.

2. Use the fact $\lim_{h\to 0} \frac{e^h - 1}{h} = \underline{\ 1\ }$ to obtain formulas for $\frac{d}{dx}(e^x)$ and $\frac{d}{dx}(a^x)$.

3. The position (in feet) of a particle moving on a straight line is given by the function

$$s(t) = \frac{5}{t} + t^e + 2e^t + 3^t.$$

Find an expression for the (instantaneous) velocity $v(t)$. What is the velocity of the particle when $t = \ln 2$ seconds?

1 parallel lines have the same slope & the derivative is the slope of the tangent line ⇒ we want to find x such that $f'(x)=m$ where m is the slope of the given line.

Step 1: Find m
↳ rewrite as $y=mx+b$

$24x-2y=3$

$-2y=3-24x$

$y=\frac{3-24x}{-2}$

$y=-\frac{3}{2}+12x$

$y=12x-\frac{3}{2}$

$m=12$

Step 2: Find $f'(x)$
↳ use power rule

$f(x)=x^3+2$

$f'(x)=3x^2+0$

$f'(x)=3x^2$

Step 3: Set $f'(x)=m$
↳ solve for x

$3x^2=12$

$x^2=4$

$x=\sqrt{4}$

$x=\pm 2$

We have now found the x values such that the slope of the tangent line is equal to the slope of the given line

Step 4: Find tangent lines w/ each slope
↳ tangent line: $y-f(x_1)=f'(x_1)(x-x_1)$

case a: $x_1=2$

$f(2)=(2)^3+2$

$=8+2$

$=10$

$y-10=12(x-2)$

$y-10=12x-24$

$y=12x-14$

case b: $x_2=-2$

$f(-2)=(-2)^3+2$

$=-8+2$

$=-6$

$y-(-6)=12(x-(-2))$

$y+6=12(x+2)$

$y+6=12x+24$

$y=12x+18$

2. Use the fact $\lim_{h\to 0} \frac{e^h - 1}{h} = \underline{\ 1\ }$ to obtain formulas for $\frac{d}{dx}(e^x)$ and $\frac{d}{dx}(a^x)$.

$\frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

(a) Find $\frac{d}{dx}(e^x)$

$$\frac{d}{dx}(e^x) = \lim_{h\to 0} \frac{e^{x+h}-e^x}{h}$$
$$= \lim_{h\to 0} \frac{e^h e^x - e^x}{h}$$
$$= \lim_{h\to 0} \frac{e^x(e^h-1)}{h}$$
$$= \lim_{h\to 0} e^x\left(\frac{e^h-1}{h}\right)$$
$$= e^x \cdot \lim_{h\to 0}\left(\frac{e^h-1}{h}\right)$$
$$= e^x(1) = e^x$$

(b) Find $\frac{d}{dx}(a^x)$

$$\frac{d}{dx}(a^x) = \lim_{h\to 0} \frac{a^{x+h}-a^x}{h}$$
$$= \lim_{h\to 0} \frac{a^h a^x - a^x}{h}$$
$$= \lim_{h\to 0} \frac{a^x(a^h-1)}{h}$$
$$= \lim_{h\to 0} a^x\left(\frac{a^h-1}{h}\right)$$
$$= a^x \cdot \lim_{h\to 0}\left(\frac{a^h-1}{h}\right)$$
$$= a^x \cdot \ln(a)$$

$\lim_{h\to 0}\left(\frac{a^h-1}{h}\right) = \ln(a)$ is a "known"

3. The position (in feet) of a particle moving on a straight line is given by the function

$$s(t) = \frac{5}{t} + t^e + 2e^t + 3^t.$$

Find an expression for the (instantaneous) velocity $v(t)$. What is the velocity of the particle when $t = \ln 2$ seconds?

instantaneous velocity: $\lim_{h\to 0} \frac{s(t+h)-s(t)}{h} = s'(t)$

using derivative rules: $\frac{d}{dx}(ax^n) = a\cdot n x^{n-1}$, $\frac{d}{dx}(ae^x) = ae^x$, $\frac{d}{dx}(a^x) = a^x \cdot \ln(a)$

$$s(t) = 5t^{-1} + t^e + 2e^t + 3^t$$
$$s'(t) = -5t^{-1-1} + et^{e-1} + 2e^t + 3^t \cdot \ln(3)$$
$$= -5t^{-2} + et^{e-1} + 2e^t + 3^t \ln(3)$$

$$s'(\ln(2)) = -5(\ln(2))^{-2} + e[\ln(2)]^{e-1} + 2e^{\ln(2)} + 3^{\ln(2)} \cdot \ln(3)$$
$$= \frac{-5}{[\ln(2)]^2} + e[\ln(2)]^{e-1} + 4 + 3^{\ln(2)} \ln(3)$$