Math 10350 – Example Set 06C
Section 3.8 Implicit Differentiation
including Logarithmic Differentiation

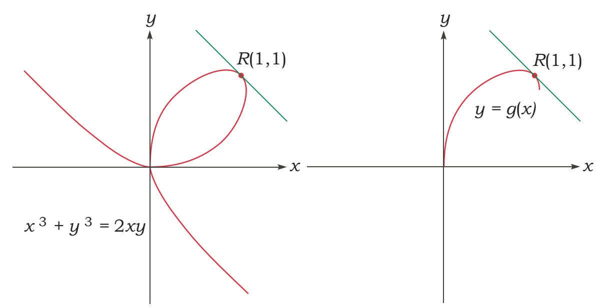
1. Find the derivative of the given functions:

(a) $(2x+1)^{\cos(e)}$ (b) $(2e+1)^{\cos x}$ (a) $(2x+1)^{\cos x}$

2. Find the equation of the tangent line at the point $P(1,2)$ on the circle $x^2+y^2=5$ by solving for y as an appropriate expression of x.

Remark: For a general relation between x and y, it is difficult to write y as a function of x. For example, $x^3+y^3=2xy$. To find the slope at $R(1,1)$ on the curve using the above method, we need to find **explicitly** $g(x)$. This is very hard!!

We say that y is an implicit function of x. To find $\frac{dy}{dx}$ in such situation we employ a powerful method called **Implicit Differentiation**.



3. Verify that the point $(1,1)$ is on the curve $x^3+y^3=2xy$. Find (a) $\frac{dy}{dx}$, (b) the slope of the curve at $(1,1)$, and (c) the point(s) on the curve where the tangent line is horizontal. (b) $y=-x+2$; (c) $\left(\frac{16^{1/3}}{3}, \frac{16^{2/3}}{6}\right)$

4. Find $\frac{dy}{dx}$ if $\cos(xy)=x+y^2$.

1(a) $y = \underbrace{(2x+1)}_{\text{chain rule}}^{\cos(e)}$ this is a constant ⇒ power rule

$y' = (\cos(e)) \cdot (2x+1)^{\cos(e)-1} \cdot (2)$

(b) $y = \underbrace{(2e+1)}_{\text{this is a constant}}^{\overline{\cos x}\ \text{chain rule}}$

⇒ $y = a^x$; $y' = a^x \ln a$

$y' = (2e+1)^{\cos(x)} \cdot \ln(2e+1) \cdot [-\sin(x)]$

$= -\sin(x) \cdot \ln(2e+1) \cdot (2e+1)^{\cos x}$

(c) $y = \underbrace{(2x+1)}_{\text{chain}}^{\underline{\cos x}\ \text{chain}}$

we don't know how to derive $y = x^x$

$y = (2x+1)^{\cos x}$

$\ln(y) = \ln((2x+1)^{\cos x})$

$e^{\ln(y)} = e^{\cos x \cdot \ln(2x+1)}$

$y = e^{\overbrace{\underbrace{\cos x \cdot \ln(2x+1)}_{\text{chain + product}}}^{\text{chain}}}$

now we can derive

$y' = e^{\cos x \cdot \ln(2x+1)} \cdot \left[(-\sin x)\ln(2x+1) + \frac{2}{2x+1} \cdot \cos x\right]$

2. Find the equation of the tangent line at the point $P(1,2)$ on the circle $x^2+y^2=5$ by solving for y as an appropriate expression of x.

$x^2+y^2=5$

we need to find y'

$y^2=5-x^2$

$y=\pm\sqrt{5-x^2}$ P(1,2) lies in the upper hemisphere

$y=\sqrt{5-x^2}=(5-x^2)^{1/2}$

$y'=\frac{1}{2}(5-x^2)^{-1/2}\cdot(-2x)$

$y'=-x(5-x^2)^{-1/2}=-\frac{x}{\sqrt{5-x^2}}$

we need to find $f'(x_1)$:

$y'(1)=-\frac{(1)}{\sqrt{5-(1)^2}}$

$=-\frac{1}{\sqrt{4}}$

$=-\frac{1}{2}$

tangent line:

$y-f(x_1)=f'(x_1)(x-x_1)$

$y-2=-\frac{1}{2}(x-1)$

$y-2=-\frac{1}{2}x+\frac{1}{2}$

$+2 \qquad +4/2$

$y=-\frac{1}{2}x+\frac{5}{2}$

3. Verify that the point $(1,1)$ is on the curve $x^3+y^3=2xy$. Find (a) $\dfrac{dy}{dx}$, (b) the slope of the curve at $(1,1)$, and (c) the point(s) on the curve where the tangent line is horizontal.

(b) $y=-x+2$; (c) $\left(\frac{16^{1/3}}{3},\frac{16^{2/3}}{6}\right)$

3. verify (1,1) is on the curve

$x^3+y^3=2xy$

$(1)^3+(1)^3=2(1)(1)$

$1+1=2$

$2=2$ ✓

(a) Find $\frac{dy}{dx}$

$x^3+y^3=2xy$

$3x^2+3y^2\cdot y'=(2)(y)+(y')(2x)$

$3x^2+3y^2y'=2y+2xy'$

$-3x^2 \quad -2xy' \quad -3x^2 \quad -2xy'$

$3y^2y'-2xy'=2y-3x^2$

$y'(3y^2-2x)=2y-3x^2$

$y'=\frac{2y-3x^2}{3y^2-2x}$

(b) Slope at (1,1)

$y'=\frac{2(1)-3(1)^2}{3(1)^2-2(1)}$

$=\frac{2-3}{3-2}$

$=\frac{-1}{1}$

$=-1$

(c) Horizontal slope

$0=y'=\frac{2y-3x^2}{3y^2-2x}$

$0=2y-3x^2$

$3x^2=2y$

$\frac{3}{2}x^2=y$ slope is 0

warning! the point must satisfy this & be on the curve

ex. $(1,\frac{3}{2})$ is not on the curve

$x^3+(\frac{3}{2}x^2)^3=2x(\frac{3}{2}x^2)$ satisfies curve

$x^3+\frac{27}{8}x^6=3x^3$

$0=2x^3-\frac{27}{8}x^6$

$0=x^3(2-\frac{27}{8}x^3)$

$x=0 \qquad 2-\frac{27}{8}x^3=0$

$2=\frac{27}{8}x^3$

$\frac{16}{27}=x^3$

$(\frac{16}{27})^{1/3}=x$

$x=0,\frac{16^{1/3}}{3} \Rightarrow y=0,\frac{16^{2/3}}{6}$

4. Find $\dfrac{dy}{dx}$ if $\cos(xy)=x+y^2$.

$\cos(xy)=x+y^2$

implicit differentation:

$-\sin(xy)\cdot[(1)(y)+(y')(x)]=1+2y\cdot y'$

$-y\sin(xy)-xy'\sin(xy)=1+2yy'$

$+y\sin(xy) \quad -2yy' \qquad +y\sin(xy) \quad -2yy'$

$-2yy'-xy'\sin(xy)=1+y\sin(xy)$

$y'(-2y-x\sin(xy))=1+y\sin(xy)$

$y'=\frac{1+y\sin(xy)}{-2y-x\sin(xy)}$

Math 10350 – Example Set 06C
Power Functions, Exponential Functions, and Mixing them.

1. Determine whether the following functions are of the form $\underset{\text{power}}{[f(x)]^n}$, $\underset{\text{exponential}}{a^{g(x)}}$, and $\underset{\text{implicit}}{[f(x)]^{g(x)}}$ where a and n are constants, and $f(x)$ and $g(x)$ are functions of x. Find their derivatives.

a. $y = (\underbrace{2x^2+5}_{\text{has x}})^{\underline{e^2+3}}$ constant $[f(x)]^n$

$y' = (e^2+3)(2x^2+5)^{e^2+2}$

b. $y = (\underbrace{2x^2+5}_{\text{has x}})^{\underline{e^x+3}}$ has x $[f(x)]^{g(x)}$

$\ln(y) = \ln((2x^2+5)^{e^x+3})$

$\ln(y) = (e^x+3)\cdot\ln(2x^2+5)$

$\frac{1}{y}\cdot y' = (e^x)\cdot\ln(2x^2+5) + \left(\frac{4x}{2x^2+5}\right)(e^x+3)$

$y' = y\cdot\left[e^x\ln(2x^2+5) + (e^x+3)\left(\frac{4x}{2x^2+5}\right)\right]$

you can plug in $y = (2x^2+5)^{e^x+3}$

c. $y = (\underbrace{2\pi^2+5}_{\text{constant}})^{\underline{x^2+3}}$ has x $a^{g(x)}$

$y' = (2\pi^2+5)^{x^2+3}\cdot\ln(2\pi^2+5)\cdot(2x)$ chain rule

d. $y = (\underbrace{\sin(e)+\cos^2(e)}_{\text{constant}})^{\underline{x^2}}$ has x $a^{g(x)}$

$y' = (\sin(e)+\cos^2(e))^{x^2}\cdot\ln(\sin(e)+\cos^2(e))\cdot(2x)$ chain rule

e. $y = (\underbrace{\sin(e)+\cos^2(e)}_{\text{constant}})^{\underline{\pi^2}}$ constant a^n

$y' = 0$ derivative of a constant is always <u>zero</u>

f. $y = (\underbrace{\sin(x)+\overbrace{\cos^2(e)}^{\text{constant}}}_{\text{has x}})^{\underline{e^2}}$ constant $[f(x)]^n$

$y' = e^2(\sin(x)+\cos^2(e))^{e^2-1}\cdot[\cos(x)]$ chain rule