Math 10350 – Example Set 07B
3.10 Related Rates
Section 11.1 Parametric Equations (Application of Chain Rule)

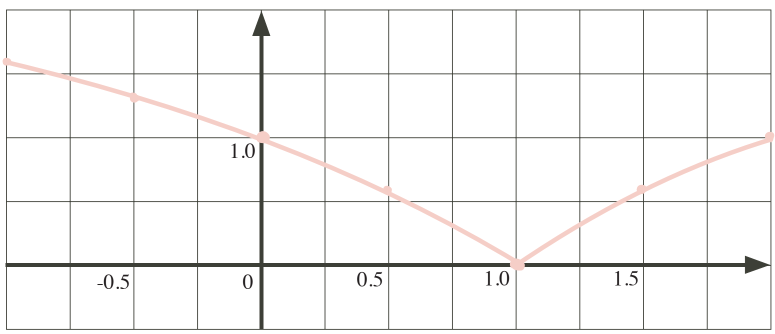
1. Water is leaking out of a conical tank with pointed end down at the rate of 9 ft^3/min. The tank is 4 feet high and the radius at the top is 3 feet. At what rate is the water level changing when the water is 2.5 feet deep?

2. A 5 meter long ladder leaning against a vertical wall such that one end is on the wall and the other end is on the ground about 0.5 meter away. If the top end of the ladder is slipping down the wall at a constant rate of 1/4 meter/min, how fast is the lower end of the ladder on the ground moving away from the wall when the lower end is 3 meters from the bottom of the wall?

3a. (Section 11.1) Find the coordinates at the time $t = 0, 0.5, 1, 1.5, 2, 2.5, 3$ of a particle following the path given by the **parametric equations:** $x = t - 1$; $y = (t-2)^{2/3}$

t	0.0	0.5	1.0	1.5	2.0	2.5	3.0
x	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
y	$(-2)^{2/3}=1.6$	$(-\frac{3}{2})^{2/3}=1.3$	$(-1)^{2/3}=1$	$(-\frac{1}{2})^{2/3}=0.6$	0	$(\frac{1}{2})^{2/3}=0.6$	1

3b. Plot the points in (a) and draw the curve given by the parametric equations. Indicate the direction of your path.



3c. By eliminating the parameter, find the cartesian equation for the path you drew.

solve one equation for t & substitute

$x = t-1 \Rightarrow x+1 = t$

$y = (t-2)^{2/3}$

$y = ((x+1)-2)^{2/3}$

$= (x-1)^{2/3}$

Remark: Using Chain Rule, obtain the slope formula $\dfrac{dy}{dx}$ for the parametric equations $x = f(t)$ and $y = g(t)$.

chain rule:

$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

$$\boxed{\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \underline{\qquad\qquad}}$$

3d. Find the equation of the tangent line to the path at $t = 10$.

$y = (x-1)^{2/3}$

$\frac{dy}{dt} = \frac{2}{3}(x-1)^{-1/3} \cdot \frac{dx}{dt}$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2}{3}(x-1)^{-1/3}$

$x(t) = t-1$

$x(10) = 10-1$

$= 9$

$y = (t-2)^{2/3}$

$y(10) = (10-8)^{2/3}$

$= \sqrt[3]{(2)^2}$

$= 4^{1/3}$

$y' = \frac{2}{3}(x-1)^{-1/3}$

$= \frac{2}{3}(9-1)^{-1/3}$

$= \frac{2}{3}(8)^{-1/3}$

$= \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$

$y - 4^{1/3} = \frac{1}{3}(x-9)$

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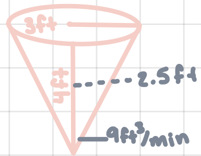

Given: rate of change of volume $\frac{dV}{dt} = -9 ft^3/min$

asked for: rate of chance of water level $\frac{dh}{dt}$ at 2.5m

Step 1: find relations

$V = \frac{1}{3}\pi r^2 h$

$\frac{r}{h} = \frac{3}{4} \Rightarrow r = \frac{3}{4}h$

$V = \frac{1}{3}\pi\left(\frac{3}{4}h\right)^2 h$

$= \frac{1}{3}\pi \cdot \frac{9}{16}h^2 \cdot h$

$= \frac{3}{16}\pi h^3$

Step 2: implicitly derive

$\frac{dV}{dt} = \frac{3}{16}\pi \cdot 3h^2 \cdot \frac{dh}{dt}$

$= \frac{9}{16}\pi h^2 \cdot \frac{dh}{dt}$

Step3: plug in knowns

$\frac{dV}{dt} = \frac{9}{16}\pi h^2 \frac{dh}{dt}$

$\cdot\frac{16}{9\pi}\; -9 = \frac{9}{16}\pi(2.5)^2 \frac{dh}{dt} \;\cdot\frac{16}{9\pi}$

$-\frac{16}{\pi} = \left(\frac{5}{2}\right)^2 \cdot \frac{dh}{dt}$

$\frac{4}{25}\cdot -\frac{16}{\pi} = \frac{25}{4}\cdot\frac{dh}{dt} \cdot \frac{4}{25}$

$-\frac{64}{25\pi} = \frac{dh}{dt}$

2. A 5 meter long ladder leaning against a vertical wall such that one end is on the wall and the other end is on the ground about 0.5 meter away. If the top end of the ladder is slipping down the wall at a constant rate of 1/4 meter/min, how fast is the lower end of the ladder on the ground moving away from the wall when the lower end is 3 meters from the bottom of the wall?

Given: h= 5m (always), rate of change of y $\frac{dy}{dt} = \frac{-1}{4}$ m/s

asked for: rate of change of x $\frac{dx}{dt}$

Step 1: find relations

$x^2 + y^2 = h^2$

$x^2 + y^2 = 5^2 = 25$

Step 2: implicitly derive

$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$

Step3: Knowns

$2(3)\cdot\frac{dx}{dt} + 2(4)\cdot\left(-\frac{1}{4}\right) = 0$

$6\frac{dx}{dt} = -2$

$\frac{dx}{dt} = -\frac{1}{3}$