

Math 10350 – Example Set 08A

Let  $f(x)$  be differentiable at  $x = a$ . Then the linear approximation for the change  $\Delta f$  in  $f(x)$  when  $x$  changes from  $a$  to  $a + \Delta x$ :

$$\Delta f = f(a + \Delta x) - f(a) \approx f'(a)\Delta x$$

1. The diameter of a circular disk is given as 10 cm with a maximum error in measurement of 0.2 cm. Use linear approximation to estimate the maximum error ( $\Delta A$ ) and percentage error in the calculated area of the disk. If the disk is made with an expensive titanium sheet that costs \$50 per  $\text{cm}^2$ , estimate an upper limit for your budget in making a disk of 10 cm diameter (Upper limit for budget is  $\$(1250\pi + 50\pi) = \$1300\pi$ ).

2. A vessel is in the shape of an inverted cone. The radius of the top is 5 cm and the height is 8 cm. Water is poured in to a height of  $x$  cm. Find an expression for the volume  $V$  of the water in the vessel in terms of  $x$ . Use linear approximate to estimate the increased in  $V$  when  $x$  increases from 4 cm to 4.08 cm. Give units for your answer. (Answer:  $\pi/2 \text{ cm}^3$ )

1.  $d = 10 \pm 0.2$  find max  $\Delta A$  & percent error =  $100 \cdot \frac{\text{approx} - \text{exact}}{\text{exact}} = 100 \cdot \frac{\Delta f}{f}$   
 & find cost to make  $C(A) = 50 \cdot A$  for largest disk

$A = \pi r^2$        $a = 10$     $\Delta r = \pm 0.1$       careful: 0.2 is diameter error

$\Delta A = A'(a) \cdot \Delta r \longrightarrow \Delta A = 10\pi(\pm 0.1)$

percent:  $100 \cdot \frac{\Delta A}{A} = 100 \cdot \frac{\pm \pi}{25\pi} = \pm 4$

$A = \pi r^2$   
 $A' = 2\pi r$   
 $A'(5) = 10\pi$        $\Delta A = \pm \pi$

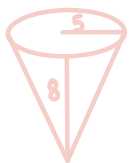
careful:  
 diam. = 10cm  
 radius = 5cm

plan to make a disk of area  $A + \Delta A$

$C(A + \Delta A) = 50(A + \Delta A) = 50A + 50\Delta A$   
 $= 50(25\pi) + 50(\pi)$   
 $= 1250\pi + 50\pi$   
 $= 1300\pi$

Two ways to compute:  
 (i) change givens to radius  
 (ii) change area to diameter

2. Given  $\frac{r}{h} = \frac{5}{8}$ . Find  $V(x)$  and  $\Delta V$  when  $\Delta x = 4.08 - 4$ .



$V = \frac{1}{3} \pi r^2 h$

use  $r = \frac{5}{8} h$

$= \frac{1}{3} \pi \left(\frac{5}{8} h\right)^2 \cdot h$

$= \frac{1}{3} \pi \cdot \left(\frac{25}{64} h^2\right) \cdot h$

$= \frac{25}{192} \pi h^3$

in terms of  $x$ :

$V(x) = \frac{25}{192} \pi x^3$

revert back to  $= \frac{1}{3} \cdot \frac{25}{64} \pi x^3$

$V'(x) = \frac{25}{64} \pi x^2$

$V'(4) = \frac{25}{64} \pi (4)^2 = \frac{25}{4} \pi$

$\Delta V = V'(x_i) \Delta x$

$\Delta V = \frac{25}{4} \pi (x - 4)$

$0.08 = \frac{8}{100}$

$= \frac{4}{50}$

for  $x = 4.08$ :

$\Delta V = \frac{25}{4} \pi (4.08 - 4)$

$= \frac{25}{4} \pi (0.08) = 0.5\pi$