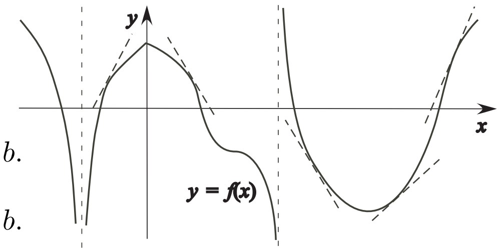
Section 4.3: First Derivative and Monotonicity

Consider the graph of $f(x)$ below.



What does $f'(x)$ tell us about the graph of $f(x)$?

(1) If $f'(x) > 0$ for $a < x < b$, then $f(x)$ is <u>increasing</u> for $a < x < b$.

(2) If $f'(x) < 0$ for $a < x < b$, then $f(x)$ is <u>decreasing</u> for $a < x < b$.

Remark: The only possible places (of x) where $f'(x)$ changes signs are at (i) <u>$f'(x)=0$</u> or at (ii) where the graph has a <u>discontinuity</u> or undefined.

> **The First Derivative Test**
> Suppose $f(x)$ has a critical point at $x = c$. We classify the critical point as follows:
> • if $f'(x)$ changes its sign from positive to negative at $x = c$, then there is a relative (local) <u>maximum</u> at $x = c$.
> • if $f'(x)$ changes its sign from negative to positive at $x = c$, then there is a relative (local) <u>minimum</u> at $x = c$.
> • if $f'(x)$ does not change its sign on both sides of $x = c$, then there is neither a relative (local) minimum nor a relative (local) maximum at $x = c$.

1. Find all values of x for which $f(x) = \frac{3}{4}x^{2/3} - x$ is increasing or decreasing with the steps outlined below. Classify all critical points using first derivative test.

Step 1: Find all **critical points** of f. (That is all points c <u>in the domain</u> where $f'(c) = 0$ or $f'(c)$ does not exist.)

$f(x) = \frac{3}{4}x^{2/3} - x$

$f'(x) = \frac{3}{4}\cdot\frac{2}{3}x^{-1/3} - 1$

$= \frac{1}{2\sqrt[3]{x}} - 1$

$f'(x)$ DNE $x=0$

$0 = \frac{1}{2\sqrt[3]{x}} - 1$

$1 = \frac{1}{2\sqrt[3]{x}}$

$\sqrt[3]{x} = \frac{1}{2}$

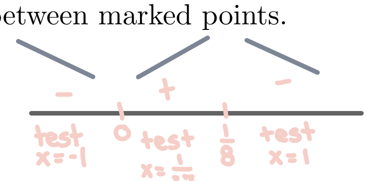
$x = \frac{1}{8}$

vertical asymptotes appear where you divide by zero

note: $f(x) = \frac{3}{4}\sqrt[3]{x^2} - x$ so there are no domain issues

Step 2: Find points where f have a **vertical asymptote** or undefined. Answer: <u>$x=0$</u>

Step 3: Draw a number line, mark all points found in Steps 1 and 2, and find the sign of $f'(x)$ in each intervals between marked points.



$f'(-1) = \frac{1}{2\sqrt[3]{-1}} - 1$
$= \frac{1}{2(-1)} - 1$
$= -\frac{1}{2} - 1$
$= -\frac{3}{2}$
negative

$f'(\frac{1}{27}) = \frac{1}{2\sqrt[3]{1/27}} - 1$
$= \frac{1}{2\cdot 1/3} - 1$
$= \frac{1}{2/3} - 1$
$= \frac{1}{1}\cdot\frac{3}{2} - 1$
$= \frac{1}{2}$ positive

$f'(1) = \frac{1}{2\sqrt[3]{1}} - 1$
$= \frac{1}{2} - 1$
$= -\frac{1}{2}$
negative

Step 4: Write down the values of x for which f is increasing ($f'(x) > 0$) and those for which f is decreasing ($f'(x) < 0$).

f is increasing on $(0, \frac{1}{8})$

this symbol means union (aka "and")

f is decreasing on $(-\infty, 0) \cup (\frac{1}{8}, \infty)$

these should always be open intervals (parenthesis) as at the endpoints $f'(x)=0$ or DNE

Step 5: Classify all critical points using first derivative test.

$x=0$ is a minimum
$x=\frac{1}{8}$ is a maximum