Math 10350 – Example Set 11C

1. Find the equations of all vertical and horizontal asymptotes of $y = \dfrac{3x^2+2x-5}{2x^2+x-3}$.

2. Sketch the graph of $f(x) = \dfrac{e^x+1}{e^x-1}$ by completing the steps below.

$f(x)=0$ $\quad$ $x=0$

a. Find all x-intercepts and y-intercept of the graph of $f(x)$ whenever possible.

$0 = \dfrac{e^x+1}{e^x-1}$

$0 = e^x+1$

$-1 = e^x$

never

$y = \dfrac{e^0+1}{e^0-1}$

$= \dfrac{1+1}{1-1}$

$= \dfrac{2}{0}$ undefined

no intercepts

b. Find coordinates of all critical points, vertical asymptotes, and places where $f(x)$ are undefined. $\left(f'(x) = -\dfrac{2e^x}{(e^x-1)^2}\right)$

$f'(x)=0$ or DNE $\quad$ denom. $=0$ $\quad$ domain issues

$f(x) = \dfrac{e^x+1}{e^x-1}$

$f'(x) = \dfrac{e^x(e^x-1)-e^x(e^x+1)}{(e^x-1)^2}$

$= \dfrac{\cancel{e^{2x}}-e^x-\cancel{e^{2x}}-e^x}{(e^x-1)^2}$

$= \dfrac{-2e^x}{(e^x-1)^2}$

DNE when $x=0$

$0 = \dfrac{-2e^x}{(e^x-1)^2}$

$0 = -2e^x$

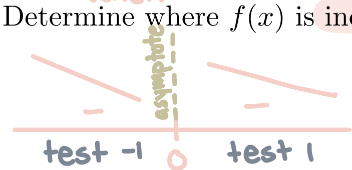
never

vertical asymptote:

$x=0$

$f'(x)>0$ $\quad$ $f'(x)<0$

c. Determine where $f(x)$ is increasing and where it is decreasing.



$f'(-1) = \dfrac{-2e^{-1}}{(e^{-1}-1)^2}$ $\quad \dfrac{\text{negative}}{\text{positive}}$ ← $(\sim)^2$ is always positive

$f'(1) = \dfrac{-2e^1}{(e^1-1)^2}$ $\quad \dfrac{\text{negative}}{\text{positive}}$

decrasing: $(-\infty,0)\cup(0,\infty)$

d. Determine the concavity and coordinates of inflection points of $f(x)$. $\left(f''(x) = \dfrac{2e^x(1+e^x)}{(e^x-1)^3} = \dfrac{2e^x(1+e^x)}{(e^x-1)^2}\cdot\dfrac{1}{e^x-1}\right)$

$f'(x) = \dfrac{-2e^x}{(e^x-1)^2}$

$f''(x) = \dfrac{-2e^x(e^x-1)^2 - 2(e^x-1)\cdot(e^x)(-2e^x)}{((e^x-1)^2)^2}$

$= \dfrac{-2e^x(e^x-1)^2+4e^{2x}(e^x-1)}{(e^x-1)^4}$

$= \dfrac{-2e^x(e^x-1)+4e^{2x}}{(e^x-1)^3}$

$= \dfrac{-2e^{2x}+2e^x+4e^{2x}}{(e^x-1)^3}$

$= \dfrac{2e^{2x}+2e^x}{(e^x-1)^2}$

$= \dfrac{2e^x(e^x+1)}{(e^x-1)^2}$

$0 = \dfrac{2e^x(e^x+1)}{(e^x-1)^2}$

$0 = 2e^x(e^x+1)$

$0 = 2e^x$ $\quad$ $0 = e^x+1$

never $\quad$ $-1 = e^x$

never

DNE when denom. $=0$

$(e^x-1)^4 = 0$

$e^x-1=0$

$e^x = 1$

$x=0$

$f''(-1) = \dfrac{(2e^{-1})(e^{-1}+1)}{(e^{-1}-1)^3}$ $\quad \dfrac{\text{pos.} \times \text{pos.}}{(\text{neg.})^3}$

$f''(1) = \dfrac{(2e^1)(e^1+1)}{(e^1+1)^3}$ $\quad \dfrac{\text{pos.} \times \text{pos.}}{(\text{pos.})^3}$

concave up: $(0,\infty)$ $\quad$ concave down: $(-\infty,0)$

1. Find the equations of all vertical and horizontal asymptotes of $y = \dfrac{3x^2+2x-5}{2x^2+x-3}$.

denominator $=0$

$\lim_{x\to\pm\infty} f(x) = L$

vertical:

$2x+3=0$

$2x=-3$

$x=-3/2$

remove all holes:

$y = \dfrac{(3x+5)(x-1)}{(x-1)(2x+3)} = \dfrac{3x+5}{2x+3}$

hole at $x=1$

horizontal:

$\lim_{x\to\pm\infty} \dfrac{3x+5}{2x+3}$ $\quad \frac{\infty}{\infty}$ L'H

$= \lim_{x\to\pm\infty} \dfrac{3}{2}$

$= \dfrac{3}{2}$

$y = \dfrac{3}{2}$