Math 10350 – Example Set 12C

1. Find the dimensions of a **closed** cylindrical can with maximum volume if the surface area is 200π cm^2.

At least set up the following optimization problems. Be sure to give the domain for the optimization problem.

2. A 100 meter long wire is to be bent into a shape consisting of a semi-circular side and two equal straight sides. Find the dimensions of the shape if the area enclosed is to be maximized. Your answer should give the radius of the semi-circular side and the length of the two equal straight sides.

3. Consider a shape consisting of a semi-circular side and two equal straight sides which encloses an area of 100 sq. meters. Find the dimensions of the shape if its perimeter is to be shortest. Your answer should give the radius of the semi-circular side and the length of the two equal straight sides.

1.

V = (area circle) × (height)

S.A. = (circumference) × (height) + 2(area circle)

Domain: $0 < r \leq 10$

$200\pi = 2\pi r^2$

$100 = r^2$

$\pm 10 = r$

$h = 0$

$\Rightarrow V = 0$

Given: $200\pi = (2\pi r)(h) + 2\pi r^2 \longrightarrow \dfrac{200\pi - 2\pi r^2}{2\pi r} = h$

$V = (\pi r^2)(h)$

$V = (\pi r^2)\left(\dfrac{100 - r^2}{r}\right)$

$= r(100 - r^2)$

$= 100r - r^3$

Maximize Volume

$\dfrac{dV}{dr} = 100 - 3r^2 = 0$

$100 = 3r^2$

$\dfrac{100}{3} = r^2$

$\sqrt{\dfrac{100}{3}} = r$ throw out negative r

$\dfrac{10}{\sqrt{3}} = r$

$h = \dfrac{100 - \left(\sqrt{\frac{100}{3}}\right)^2}{\left(\frac{100}{3}\right)^{1/2}}$

$= \dfrac{100 - \frac{10}{\sqrt{3}}}{\frac{10}{\sqrt{3}}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$

$= \dfrac{100\sqrt{3} - 10}{10}$

$= 10\sqrt{3} - 1$

pos. when $r < \frac{10}{\sqrt{3}}$

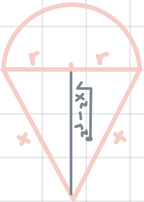
neg. when $r > \frac{10}{\sqrt{3}}$

maximum volume

Remember h=0 still satisfies the constraint even if it makes the volume zero.

You are expected to set up but not solve these problems

Question 2 & 3 address the same shape.



Area = (triangle) + $\frac{1}{2}$(circle)
$= \frac{1}{2}(2r)(x^2-r^2)^{1/2} + \frac{1}{2}(\pi r^2)$
$= r(x^2-r^2)^{1/2} + \frac{1}{2}\pi r^2$

Perimeter = (two sides) + $\frac{1}{2}$(circumference)
$= 2x + \frac{1}{2}(2\pi r)$
$= 2x + \pi r$

2. Given Perimeter $= 2x + \pi r = 100 \longrightarrow x = \frac{100-\pi r}{2}$
Area $= r(x^2-r^2)^{1/2} + \frac{1}{2}\pi r^2$ $\qquad A = r\left(\left(\frac{100-\pi r}{2}\right)^2 - r^2\right)^{1/2} + \frac{1}{2}\pi r^2$

Simplify Area
$r\left[\frac{1}{4}(100-\pi r)^2 - r^2\right]^{1/2} + \frac{1}{2}\pi r^2$
$r\left[\frac{1}{4}(10000 - 200\pi r + \pi^2 r) - r^2\right]^{1/2} + \frac{1}{2}\pi r^2$
$r\left[2500 - 50\pi r + \pi^2 r^2 - r^2\right]^{1/2} + \frac{1}{2}\pi r^2$
$\left[r^2(2500 - 50\pi r + \pi^2 r^2 - r^2)\right]^{1/2} + \frac{1}{2}\pi r^2$
$\left[2500r^2 - 50\pi r^3 + \pi^2 r^4 - r^4\right]^{1/2} + \frac{1}{2}\pi r^2$

domain: $0 \le r \le \frac{100}{\pi}$
just a line semi circle is a point
just a semi-circle when $x=0$, $\pi r = 100$

Set $\frac{dA}{dr} = 0$
$\frac{1}{2}(250r^2 - 50\pi r^3 + \pi^2 r^4 - r^4)^{-1/2}(5000r - 150\pi r^2 + 4\pi^2 r^3 - 4r^3) + \pi r = 0$ using simplified

$(1)\left[\frac{1}{4}(100-\pi r)^2 - r^2\right]^{1/2} + \frac{1}{2}\left[\frac{1}{4}(100-\pi r)^2 - r^2\right]^{-1/2} \cdot \left[\frac{1}{8}(100-\pi r)\cdot(-\pi) - 2r\right](r) + \pi r^2 = 0$ using original

3. Given Area $= r(x^2-r^2)^{1/2} + \frac{1}{2}\pi r^2 = 100 \longrightarrow (x^2-r^2)^{1/2} = \frac{100-\frac{1}{2}\pi r^2}{r}$
Perimeter $= 2x + \pi r$ $\qquad x = \left(\frac{100-\frac{1}{2}\pi r^2}{r}\right)^2 + r^2$

Simplify Perimeter
$P = 2\left[\left(\frac{100-\frac{1}{2}\pi r^2}{r}\right)^2 + r^2\right] + \pi r$
$= 2r^{-2}(100-\frac{1}{2}\pi r^2)^2 + 2r^2 + \pi r$

Set $\frac{dP}{dr} = 0$
$-4r^{-3}(100-\frac{1}{2}\pi r^2)^2 + 2(100-\frac{1}{2}\pi r^2)\cdot(-\pi r)(2r^{-2}) + 4r + \pi = 0$ using simplified