Math 10350 – Example Set 15C
Sections 5.5, 5.6, & 5.7

1. (Section 5.7 Substitution) Evaluate the following integrals:

a. $\int_0^{\pi/4} \sin 4t \, dt$

We are looking for a function F(t) whose derivative is sin(4t). Our first guess might be $F(t)=\cos(t)$, but $F'(t)=-\sin(t)$. We need it to be positive so we test $F(t)=-\cos(t)$. Now $F'(t)=\sin(t)$ which is closer, but we need a 4t. So we choose $F(t)=-\cos(4t)$, thus $F'(t)=4\sin(4t)$. Final guess $F(t)=-\frac{1}{4}\cos(4t)+c$.

We speed this up with substitution:

$u=4t \Rightarrow$ $t=0, u=0$; $t=\frac{\pi}{4}, u=\pi$; $du=4dt$ $\Rightarrow \int_0^{\pi} \sin(u)\cdot\frac{1}{4}du$

$= \frac{1}{4}(-\cos(u))\Big|_0^{\pi}$

$= -\frac{1}{4}\cos(u)\Big|_0^{\pi}$

$= -\frac{1}{4}\cos(4t)\Big|_0^{\pi/4}$

b. $\int x\sqrt{2-3x}\, dx$

$u=2-3x \Rightarrow \frac{u-2}{3}=x$

$du=-3dx \Rightarrow -\frac{1}{3}du=dx$

$\int \left(\frac{u-2}{3}\right)\sqrt{u}\left(-\frac{1}{3}\right)du$

$=-\frac{1}{9}\int (u-2)u^{1/2}du$

$=-\frac{1}{9}\int u^{3/2}-2u^{1/2}\, du$

$=-\frac{1}{9}\left[\frac{2}{5}u^{5/2}-2\left(\frac{2}{3}\right)u^{3/2}+c\right]$

c. $\int_1^4 \frac{1}{x^2}\sqrt{1+\frac{1}{x}}\, dx$

From here on you should be looking for derivatives inside of integrals.

Inner function: $u=1+\frac{1}{x}$

Derivative: $du=-\frac{1}{x^2}dx$

Bounds: $x=1, u=1+\frac{1}{1}=2$

$x=4, u=1+\frac{1}{4}=\frac{5}{4}$

$\int_2^{5/4}\sqrt{u}\,du=\int_2^{5/4}u^{1/2}du=\frac{2}{3}u^{3/2}\Big|_2^{5/4}=\frac{2}{3}\left[\left(\frac{5}{4}\right)^{3/2}-(2)^{3/2}\right]$

$=\frac{2}{3}\left[\frac{5\sqrt{5}}{8}-2\sqrt{2}\right]$ ↑ I would stop here

d. $\int \theta^3\sec^2(\theta^4+1)\, d\theta$

$u=\theta^4+1$

$du=4\theta^3d\theta \Rightarrow \frac{1}{4}du=\theta^3d\theta$

$\int \sec^2(\theta^4+1)\cdot\theta^3d\theta=\int\sec^2(u)\cdot\frac{1}{4}du$

$=\frac{1}{4}\int\sec^2(u)\,du$

$=\frac{1}{4}\tan(u)+c$

e. $\int_1^2 x^2e^{x^3+2}\, dx$

$u=x^3+2$

$du=3x^2dx \Rightarrow \frac{1}{3}du=x^2dx$

$x=1, u=2$

$x=2, u=(2)^3+2=10$

$\int_2^{10}e^u\cdot\frac{1}{3}du=\frac{1}{3}e^u\Big|_2^{10}$

$=\frac{1}{3}e^{10}-\frac{1}{3}e^2$

$=\frac{1}{3}e^2(e^{10}-1)$

f. $\int \frac{t+1}{t^2+2t+5}\, dt$

$u=t^2+2t+5$

$du=2t+2\,dt \Rightarrow \frac{1}{2}du=(t+1)dt$

$\int\frac{1}{u}du=\ln(u)+c=\ln(t^2+2t+5)+c$