Exit Ticket Derivative and Integral Review

Fill in the derivatives and integrals:

1. $\frac{d}{dx}[k] =$
2. $\displaystyle\int k dx =$
3. $\frac{d}{dx}[kx^n] =$
4. $\displaystyle\int x^n dx =$
5. $\frac{d}{dx}[\ln(x)] =$
6. $\displaystyle\int \frac{1}{x} dx =$
7. $\frac{d}{dx}[\log_a(x)] =$
8. $\displaystyle\int \frac{1}{x \cdot \ln(a)} dx =$
9. $\frac{d}{dx}[e^x] =$
10. $\displaystyle\int e^x dx =$
11. $\frac{d}{dx}[a^x] =$
12. $\displaystyle\int a^x dx =$
13. $\frac{d}{dx}[\sin(x)] =$
14. $\displaystyle\int \cos(x) dx =$
15. $\frac{d}{dx}[\cos(x)] =$
16. $\displaystyle\int \sin(x) dx =$
17. $\frac{d}{dx}[\tan(x)] =$
18. $\displaystyle\int \sec^2(x) dx =$
19. $\frac{d}{dx}[\sec(x)] =$
20. $\displaystyle\int \sec(x)\tan(x) dx =$

Use the rules above to find the integrals below and check your answer:

1. $\displaystyle\int \cot(x)\sin(x) dx$
2. $\displaystyle\int \frac{1+\cos^2(\theta)}{\cos^2(\theta)} d\theta$
3. $\displaystyle\int \frac{2u^2 - 5u + \sqrt[3]{u}}{u^2} du$
4. $\displaystyle\int 6x(x^2+1)^2 dx$

Produced by Audriana Houtz, Mathematics Ph.D. student at the University of Notre Dame.

Exit Ticket Natural Log

Fill in the following rules:

1. $\ln(a) + \ln(b) =$

2. $\ln(a) - \ln(b) =$

3. $\ln(x^a) =$

4. $\ln\left(ax^b\right) =$

5. $\frac{d}{dx}\left[\ln(ax+b)\right] =$

6. $\displaystyle\int \frac{a}{ax+b}dx =$

Use the above rules to solve the following equations for x:

1. $\displaystyle\int \frac{1}{2x+5}dx$

2. $\displaystyle\int \frac{1}{x+12}dx$

3. $\displaystyle\frac{d}{dx}\left[\ln\left(\frac{1-x}{1+x}\right)\right]$

4. $\displaystyle\frac{d}{dx}\left[\ln\left(\frac{2x^2-3}{3x^3-6}\right)\right]$

5. $\displaystyle\int \frac{2x}{4x^2+12}dx$

6. $\displaystyle\int \frac{5x+7}{5x^2+14x+6}dx$

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Exit Ticket Integral Review

Solve the following integrals and identify the integral rule used:

1. $\int \cot(x)\sin(x)dx$

2. $\int \frac{1+\cos^2(\theta)}{\cos^2(\theta)}d\theta$

3. $\int \frac{\sin(x)}{1+\cos^2(x)}dx$

4. $\int 6x(x^2+1)^{\frac{1}{2}}dx$

5. $\int \sin(2x)dx$

6. $\int \frac{1}{1+\sin(\theta)}d\theta$

7. $\int \frac{3x}{(2x^2+1)^2}dx$

8. $\int \frac{3x}{(2x^2+1)}dx$

Exit Ticket Inverse Trigonometric Functions

Fill in the derivatives and integrals:

1. $\frac{d}{dx}\left[\arcsin(x)\right] =$

2. $\int \frac{1}{\sqrt{1-x^2}}dx =$

3. $\frac{d}{dx}\left[\arctan(x)\right] =$

4. $\int \frac{1}{1+x^2}dx =$

Use the rules above to find the integrals below:

1. $\int \frac{1}{1+9x^2}dx$

2. $\frac{d}{dx}\left[\arcsin\left(\frac{3}{4}x\right)\right]$

3. $\int \frac{3}{\sqrt{9-4x^2}}dx$

4. $\frac{d}{dx}\left[\arctan\left(x^2\right)\right]$

5. $\int \frac{5x+1}{4+9x^2}dx$

6. $\frac{d}{dx}\left[\arcsin(x+1)\right]$

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Exit Ticket Area Between Curves

> **Area Between curves** Assuming that $f(x) \geq g(x)$ for $a \leq x \leq b$, the area between the curves is:
>
> $$\int_a^b [f(x) - g(x)]\, dx$$

Set up but do NOT solve the integral that finds the areas bounded by the functions below:

1. $y = x^2 + 2,\ y = \sin(x),\ x = -1,\ x = 2$

2. $x = y^2 + 1,\ x = 5,\ y = -3,\ y = 3$

3. $y = \frac{1}{x+2},\ y = (x+2)^2,\ x = -\frac{3}{2},\ x = 1$

4. $x = y^2 - y - 6,\ x = 2y + 4$

Exit Ticket Volume of Solids with Uniform Cross-sections

> **Volume of Solids with Uniform Cross-sections** Consider a solid whose base is the region bounded by given function(s) with uniform cross-sections perpendicular to the x-axis. The volume of the solid is given by:
>
> $$V = \int_a^b [A(x)]\, dx$$
>
> where $A(x)$ is the area of the cross-section

Set up but do NOT solve the integral that finds the volume of the solid whose base is bounded by $y = x^2 + 2,\ y = \sin(x),\ x = -1,\ x = 2$ and has uniform cross-sections perpendicular to the x-axis in the shape of:

1. squares

2. triangles of height x^2

3. semicircles

4. rectangles of height $\sqrt{x}$

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Exit Ticket Solids of Revolution

Solids of Revolution Consider a solid formed by rotating a bounded region about a line $y = c$ with cross-sectional area functions $A(x)$, then the volume formula is

$$V = \int_a^b [A(x)]\, dx.$$

Disk method: $A(x) = \pi r^2$ where r is a function of x
Washer method: $A(x) = \pi [R^2 - r^2]$ where R, r are a functions of x
Shell method: $A(x) = 2\pi r h$ where r, h are a functions of x

Set up but do NOT solve the integral that finds the volume of the solid formed by rotating the region bounded by:

1. $y = \sqrt{x}$, $y = 3$, and the $y-$axis about the $y-$axis

2. $y = 10 - 6x + x^2$, $y = -10 + 6x - x^2$, $x = 1$, and $x = 5$ about the line $y = 8$

3. $x = y^2 - 4$, $x = 6 - 3y$ about the line $y = 8$

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Exit Ticket Work and Energy

Work and Energy Suppose that the force at any given x is given by $F(x)$, then the work done by the force in moving the object from $x = a$ to $x = b$ is given by

$$W = \int_a^b F(x)dx.$$

Set up but do NOT solve the following integral:

1. A uniform chain 10 m long weighing 30 kg lying completely at the foot of a building 50 m tall.
   (a) What is the work done against gravity to move one end to the top of the building with the rest of the chain danging free?
   (b) What is the work done to move one end only 30 m off the ground?
   (c) What is the work done to move the top end of the chain 5 meters off the ground with the rest of the chain still on the ground?

Exit Ticket Integration by Parts

> **Integration by Parts** Let $u(x)$ and $v(x)$ be two differentiable functions. Integration by parts says
>
> $$\int u dv = ux - \int v du$$

Evaluate the following integrals:

1. $\int 8xe^{6x}dx$

2. $\int 4x\cos(2-3x)dx$

3. $\int (2-x)^2\ln(4x)dx$

4. $\int \ln(x)dx$

5. $\int e^{-x}\sin(4x)dx$

6. $\int \frac{x^7}{\sqrt{x^4+1}}dx$