Applications of Exponential and Logarithmic Functions

Exponential Growth and Decay

A quantity y is said to grow or decay exponentially if the rate of change of y is proportional to the quantity of y. In other words, y(t) satisfies the differential function:

dy de = r.y

Some real life examples:

(i) exponential growth: bacteria culture, compound intrest, viral spread, etc. (ii) exponential decay: radioactive decay, drug concentration, depreciation of assets,...

We call K the rate of growth. if r>0 then we say that y is growing exponentially if r<0 then we say that y is decaying exponentially

We can also solve the differential equation such that $y = C \cdot e^{rt}$ where C is the initial value.

Doubling Time and Half Life

If y(t) is exponentially growing, then the time it takes for the initial amount to double is called the <u>doubling time</u>, i.e. the time t such that 2C=C e^{rt}.

If y(t) is exponentially decaying, then the time it takes for the initial amount to be cut in half is called the half life, i.e. the time t such that $\frac{1}{2}C = C \cdot e^{rt}$

<u>Examples</u>

1. Recent experiments on viability of the coronavirus indicates that it reduces exponentially on various surfaces. The half life of the coronavirus on glass is estimated to be about 14 hours. (a) Starting with 100% initially, find a formula in the form $A \cdot e^{rt}$ for the percentage of the virus on glass after t hours. (b) If we consider the virus no longer infectious (or viable) after it is reduced to 1% or less, estimate how long will the virus remain infectious on glass. $A = 100, r = -\frac{\ln(2)}{14}$

Reference: Aerosol and Surface Stability of SARS-CoV-2 as Compared with SARS-CoV-1, N Engl J Med April 2020 Stability of SARS-CoV-2 in different environmental conditions, Lancet April 2020.

(a)	giv	ien:	h	alf	life	=	4h	rs,	ini	tial	= 1	00	1.	i.e	(0	0,1	00)	į (14,	50)				
		as	ked	fo	r:	٦l	t);	= A	·ert	-		3	unl	no	wns	s A	,r,	t							
		Us	e in	niti	al	to	50	lve	for	r iv	niti	a1	val	ue	A:										
		۷I	0) =	A٠	er.	0																			
		10)0 =	A٠	1																				
		Us	es	sec	ona		oin	+ +	0 5	olve	f	sr	rat	e r	:										
		v(14)	= 10)0 e	r.14					us	e H	he	A	fro	m	ab	OVE	2						
		5	0 =	100)e ¹⁴	r																			

	= e	14r					left	al	war	15	bec	omo									
In	<u>ا با</u>	= Inle ¹	4r)				hen	CP.	ha	1++1	ife										
	2'):	14~	Inle'																		
	ilo	$ 2\rangle = 1$	(1)																		
	3 17 1																				
	4)=	1000	щIn	lz)·t	•			_	200	at.		ato	- do								
4		IUUE							TIED			are	- 00								
(b) ai		• • • •				6	L:		c 1	/											
	ver	· no	ion	ger		Tec			= _		·e.	YIE									
as	Lea	tori	tim	e u	one	ve	14	IS	no	Ion	ger	1701	ect	ous							
50	Ive	tor	- 11 ly	y(t) (z)·	= 1 t								_								
	1=		zlaiz)	t																	
	100	=e"		- the les	12) •1														 	 	
	In (<u>ioo</u>)=	Inle	- 161 101)															
	In	100")	= - वि	i In(z	?) · t																
	-In	(100)	<u>= - i</u>	Inla	z) · t	-		1	this	do	es i	1 01	sim	plif	4				 	 	
	14	In(100 In(2)	; = t		¢			_ ,	nor	nał	ter	hou	v m	luc	1						
	≈3	88.	days						You	u u o	ant	_it	to								
giver	: -	omb	s wa	bod	is	at	55	٪,	CNI	pre	ss I	nalf	lif	e is	5	73(>				
asked	to	r: a	ge	ot	+0	mt	o w	00	d	i.e.	T	-	h		L.	ULT.	<u>۱</u> ـ ۱	55			
First,			<u> </u>							, ,		suc		ma	1	1.)-;				
	N 57	e ne	ed o	fo	rm	ula	γŀ	;)=	A e ^r	t		suc		ma		1)-;				
2 = e'	.57	e ne 30	ed a	l for	rm	ula	ylt	;)=	A e ^r	t		suc				4)-:				
¹ = e' In(¹ 2)	.57 - In	e ne 30 (e ⁵⁷³	ed o		rm	ula	ylt	;)=	A e ^r	t		suc				4)- ;				
ュ= e' Inlź) In lz ⁻	57 = In ') =	e ne 30 (e ⁵⁷³ 5730	ed o sor)		rm	ula	ylt	;)=	Aer	e		suc				4)- (
2 = e Inl2 Inl2 -5730		e nel 30 (e ⁵⁷³ 573((2) = r	ed o sor))r		rm	ula	ylt	;)=	A e ^r)- ;				
2 = e Inl2 In l2 -5730	:57 -57 -)= 5 n	e ne(30) (e^{573}) (5730) (2) = r	ed o sor))r			ula	_ \ [t	=(;)=	Ae)- 、				
2 = e' Inl±) Inl ⁻ -5730 Y(t)=	: 100	e ne(30) (e^{573}) (573) (2) = r (2) = r	ed 0 ⁸⁰))r -		t	ula	4 [t	;)=	Ae)- 、				
2 = e' Inl2 In l2 -5730 Y(t)=	= In () = 5 In 10	e net 30 (e^{573}) 5730 (z) = r (z) = r	ed 0 ³⁰)) - - - - - - - - - - - - -		ť		- y (H		Ae												
2 = e' Inl2 In l2 -5730 Y(t)= Solve	57 57 	e ner 30 (e^{573}) (2) = r (2) = r (2) = r	200) 30 ~)) ~ 1 230 In	(Z)···	+ +				Ae												
2 = e' In (2) In (2 ⁻ -5730 Y(t)= Solve 55 =	= In) = 100	e^{-57}	20 0 30 r 1 1 1 30 In 30 In		e t t																
$\frac{1}{2} = e^{1}$ $\ln \frac{1}{2}$ $\ln \frac{1}{2}$ $-\frac{1}{5730}$ $\gamma(t) =$ 50100 55 = $\frac{55}{100} = e^{1}$	(.57) = $\ln^{1}(.57)$ = $\ln^{1}(.57)$ = $\ln^{1}(.57)$ = $\ln^{1}(.57)$ = $\ln^{1}(.57)$	$e net30(e^{573})5730(2) = r0e^{-57}e^{-57}(n(z))$	20 0 30 r) 1 1 1 30 In T		۲m + = 55																
2 = e' Inl2 In (2 -5730 y(t)= 55 = 55 = 55 = Inl55		e net 30 $1(e^{573})$ 5730 (2) = r $0e^{-57}$ e^{-57} $n(e^{-57})$	20 0 30 r) 1 30 ln T 30 ln	(Z)···	rm t 55 T																
$\frac{1}{2} = e^{1}$ $\ln (\frac{1}{2})$ $\ln (2^{-})$ $-\frac{1}{5730}$ $\gamma(t) =$ $50 = e^{1}$ $\frac{55}{100} = e^{1}$ $\ln (\frac{150}{100})$		$e net30(e^{573})(573)(2) = r(2) = r(2) = r(2) = r(2) = r(2) = r$	200) 30)) 1 230 In 230 In 230 In 1 10 12	(z). (z). (z). (z). T	t 55 T																
$\frac{1}{2} = e^{1}$ $\ln \frac{1}{2}$ $\ln \frac{1}{2}$ -5730 $\gamma(t) =$ 55 = $\frac{55}{100} = e^{1}$ $\ln \frac{55}{100}$ $\ln \frac{55}{100}$		$e net 30 1(e^{573})5730(2) = r2) = r(2) = r(3) = r$	$\frac{1}{30}$ m $\frac{1}{30}$ m $\frac{1}{30}$ m $\frac{1}{30}$ m $\frac{1}{30}$ m $\frac{1}{30}$ m $\frac{1}{2}$ m	(Z)··· (Z)··· (Z)··T	rm 			·)=													
$\frac{1}{2} = e^{1}$ $\ln \frac{1}{2}$ $\ln \frac{1}{2}$ -5730 $\gamma(t) =$ 55 = $\frac{55}{100} = e^{1}$ $\ln \frac{55}{100}$ $\ln \frac{55}{100}$ -5730	$(-57)^{-}$	e net 30 5730 2) = r 2) = r 2) = r 2 = r 2 = r 2 = r 2 = r 2 = r r r r r r r r	$\frac{1}{30}$ m $\frac{1}{30}$ m 1	(z). (z). (z). (z). (z). (z). (z). (z).	t 55 7)=													
$\frac{1}{2} = e^{1}$ $\ln (\frac{1}{2})$ $\ln (2^{-})$ -5730 $\gamma(t) =$ 55 = $\frac{55}{100} = e^{1}$ $\ln (\frac{55}{100})$ $\ln (\frac{55}{100})$ -5730 5730		e^{-57} e^{-57}	30°) 30°) 30° 1 30° 1 30° 1 1° 2 1° 2 1°	(z)	t t t			;)= ;)= ; ; ; ; ;													
$\frac{1}{2} = e^{1}$ $\ln \frac{1}{2}$ $\ln \frac{1}{2}$ -5730 $\gamma(t) =$ 55 = $\frac{55}{100} = e^{1}$ $\ln \frac{55}{100}$ $\ln \frac{55}{100} = e^{1}$ $\ln \frac{55}{100} = e^{1}$	() () () () () () () () () () () () () (e^{-57} e^{-57} e^{-57} e^{-57} e^{-57} $n(e^{-57}$ $n($	$\frac{30^{r}}{30}$	(Z)··· (Z)·· (Z)·· (Z)· (Z)· (Z)· (Z)· (**************************************			;)= ;)=													