Exit Ticket Solids of Revolution

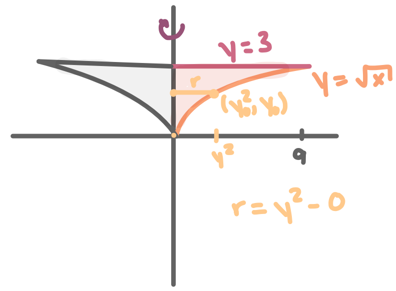
Solids of Revolution Consider a solid formed by rotating a bounded region about a line $y = c$ with cross-sectional area functions $A(x)$, then the volume formula is

$$V = \int_a^b [A(x)]\, dx.$$

Disk method: $A(x) = \pi r^2$ where r is a function of x
Washer method: $A(x) = \pi\left[R^2 - r^2\right]$ where R, r are a functions of x
Shell method: $A(x) = 2\pi r h$ where r, h are a functions of x

Set up but do NOT solve the integral that finds the volume of the solid formed by rotating the region bounded by:

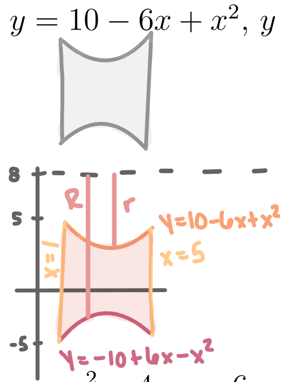
1. $y = \sqrt{x}$, $y = 3$, and the $y-$axis about the $y-$axis ← $x=0$



<u>Disk method</u>

$A(y) = \pi r^2$
$= \pi (y^2)^2$
$= \pi y^4$
$V = \int_0^3 \pi y^4 \, dy$

2. $y = 10 - 6x + x^2$, $y = -10 + 6x - x^2$, $x = 1$, and $x = 5$ about the line $y = 8$

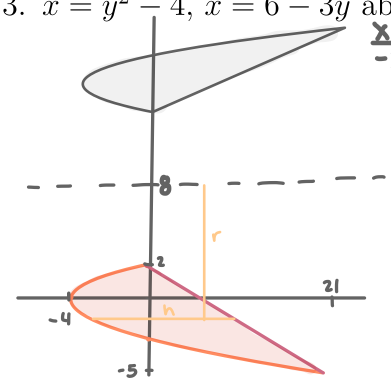


<u>washer method</u>

$A(x) = \pi(R^2 - r^2)$
$= \pi\left[(8-(-10+6x-x^2))^2 - (8-(10-6x+x^2))^2\right]$
$= \pi\left[(18-6x+x^2)^2 - (-2+6x-x^2)^2\right]$
$V = \int_1^5 \pi\left[(18-6x+x^2)^2 - (-2+6x-x^2)^2\right] dx$

3. $x = y^2 - 4$, $x = 6 - 3y$ about the line $y = 8$

$\frac{x-6}{-3} = y$



<u>shell method</u>

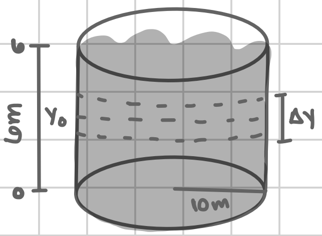
$A(y) = 2\pi r h$
$= 2\pi \underbrace{(8}_{\text{top}} - \underbrace{y)}_{\text{bottom}} (\underbrace{(6-3y)}_{\text{right}} - \underbrace{(y^2-4)}_{\text{left}})$

$V = \int_{-5}^{2} 2\pi (8-y)(10-3y-y^2)\, dy$

Work and Energy

Examples:

1. A cisten is shaped like a cylinder of height 6m and radius 10m. The circular opening of the cistern is at ground level and the rest of the cistern is buried below ground. Compute the amount of work done in pumping all the water out of the cistern from ground level if it is filled completely with water. Mass density of water is 100 kg/m³. You may take the acceleration due to gravity as $g = 10\ m/s^2$.



Amount of work of displacing Δy portion of water

$= \text{slice weight} \cdot \text{displacement}$

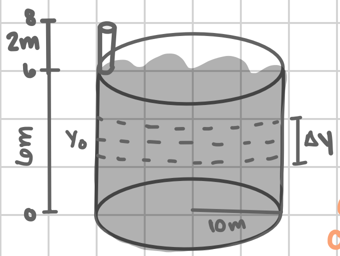
$= (\text{volume} \cdot \text{density} \cdot \text{gravity}) \cdot (6 - y_0)$

$= (\underbrace{100\pi\,\Delta y}_{\pi r^2 h})(1000)(10) \cdot (6 - y_0)$

bottom slice (y=0) moves 6m, top slice (y=6) moves 0m

$$W = \int_0^6 1000000\,\pi (6 - y)\,dy$$

(b) How would the answer change if all the water is pumped to a level 2m above the opening of the cistern?



Amount of work of displacing Δy portion of water

$= \text{slice weight} \cdot \text{displacement}$

$= (\text{volume} \cdot \text{density} \cdot \text{gravity}) \cdot (8 - y_0)$

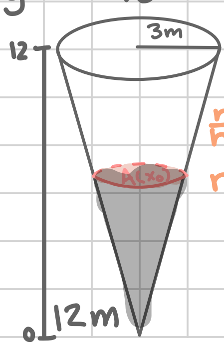
$= (100\pi\,\Delta y)(1000)(10) \cdot (8 - y_0)$

the lowest layer of water moves up 8m, the highest moves 2m

depth of water

$$W = \int_0^6 (100\pi)(1000)(10) \cdot (8 - y)\,dy$$

2. A tank is shaped like an inverted right circular cone of height 12m with circular opening of radius 3m. Assuming that the tank is filled halfway up with a certain kind of oil, compute the amount of work done in pumping all the oil to a level 2m above the opening of the tank. Density of the oil is 500 kg/m³. You may take the acceleration due to gravity as $g = 10\ m/s^2$



$\frac{r}{h} = \frac{3}{12}$

$r = \frac{1}{4}h$

$= \frac{1}{4}x$

work of displacing Δx portion of water

$= \text{slice weight} \cdot \text{displacement}$

$= \text{volume} \cdot \text{density} \cdot \text{gravity} \cdot \text{displacement}$

$= (\pi (\tfrac{1}{4}x)^2 \Delta x)(500) \cdot (10) \cdot (14 - x)$

top of water moves 6+2 m
bottom of water moves 12+2m

$$W = \int_0^6 (\tfrac{1}{16}\pi x^2)(500)(10)(14 - x)\,dx$$