## Separable Differential Equations

Differential Equations A differential equation is an equation that involves an unknown function and its first or higher derivatives. examples.  $\frac{dy}{dx} = 1 - \log^{2x}$ first order differential equations  $\frac{d_{y}}{dt} + \frac{1}{t+30} \cdot y = 4$  $\frac{d^2 y}{d^2 x} + P(x) \frac{d y}{d x} + Q(x) y = 0$  second order (homogeneous) diff. eq. dx = F(x,y)] general first order differential equations Separation of Variables Separable differential equation:  $\frac{dy}{dx} = p(x)q(y)$ examples.  $\frac{dy}{dx} = 1 - \log^{2x}$  $p(x) = 1 - be^{2x}; q(y) = 1$  $y'(x) = 3x^2y$   $p(x) = 3x^2$ ; q(y) = yHow to solve? Method of separation.  $\frac{dy}{dx} = p(x)q(y) \quad \langle = \rangle \quad y'(x) = p(x)q(y)$  $L = \frac{1}{q(Y)} + (x) = p(x)$  $<=> \int \frac{1}{q(y)} y'(x) dx = \int p(x) dx$  $\langle z \rangle \int \frac{1}{q(y)} dy = \int p(x) dx$ Simplified explanation:  $\frac{dy}{dx} = p(x)q(y)$  $\overline{q(y)} dy = p(x) dx$ 2=> Sqiyi dy = Spixi dx (=7

| E  | an          | 00         | le             | s:       |      |     |       |       |     |     |      |     |          |    |      |               |            |      |      |    |   |     |    |     |    |    |         |    |    |
|----|-------------|------------|----------------|----------|------|-----|-------|-------|-----|-----|------|-----|----------|----|------|---------------|------------|------|------|----|---|-----|----|-----|----|----|---------|----|----|
|    |             |            |                |          |      |     |       |       |     |     |      |     |          |    |      |               |            |      |      |    |   |     |    |     |    |    |         |    |    |
| 1. | So          | lve        | +              | he       | di   | ffe | ren   | tia   | le  | auc | atio | n   | dx<br>dy | =  | - Lo | ezx           |            | for  | ,    |    |   |     |    |     |    |    |         |    |    |
|    | (;)         | a          |                | solu     | atio | ons | an    | d     |     | C   |      |     |          |    |      |               |            |      |      |    |   |     |    |     |    |    |         |    |    |
|    | (11         | ) H        | ne             | 50       | ut   | ion | sat   | isf   | vin | 0   | J (0 | )=5 |          |    |      |               |            |      |      |    |   |     |    |     |    |    |         |    |    |
|    |             |            |                |          |      |     |       |       | 1   | 3   | 1.0  |     |          |    |      |               |            |      |      |    |   |     |    |     |    |    |         |    |    |
|    | 63          | dx<br>dx   | = 1            | - 10     | ezx  |     |       |       |     |     |      |     |          |    | (;;) | െ             | 11         | 0) = | 5    |    |   |     |    |     |    |    |         |    |    |
|    |             | <b>G</b>   |                |          |      |     |       |       |     |     |      |     |          |    |      |               |            |      |      |    |   |     |    |     |    |    |         |    |    |
|    |             | 5          | 4              | =        | 51-  | lne | ex d  | x     |     |     |      |     |          |    |      | 0-            | - 3e       | 0 +  | c. = | 5  |   |     |    |     |    |    |         |    |    |
|    |             |            |                |          |      |     |       | ~     |     |     |      |     |          |    |      |               |            |      |      |    |   |     |    |     |    |    |         |    |    |
|    |             | N -        |                | _ 4      | 02   | ×   | c     |       |     |     |      |     |          |    |      | _2            | 10         | _ 6  |      |    |   |     |    |     |    |    |         |    |    |
|    |             | - F        |                |          |      |     |       |       |     |     |      |     |          |    |      | -3            |            |      | /    |    |   |     |    |     |    |    |         |    |    |
|    | π           | vie        | je             | a        | m    | ar  |       | n     | ותר | PY  | 5    | hri | aci      | na |      | <b>c</b> -    | 9          |      |      |    |   |     |    |     |    |    |         |    |    |
|    | 2           | ы<br>Ц     | _13<br>00      | in       | ;+;  |     |       |       |     |     | hh   |     |          |    |      |               |            |      |      |    |   |     |    |     |    |    |         |    |    |
|    |             |            |                | Hoo      |      |     |       | कें   | ė į |     |      |     |          |    |      |               | ~ 1        | 2.   | zx , | •  |   |     |    |     |    |    |         |    |    |
|    | Υ.          |            | ر<br>لا        |          | C.   |     | 1:    |       | Ĩ Į |     |      | 170 | , 70     |    |      | <i>A</i> =    | <u>× †</u> | 30   | 1    | O  |   |     |    |     |    |    |         |    |    |
|    | +1          |            | T              | ne       | TL   |     |       | n     | -1  |     | )•   |     |          |    |      |               |            |      |      |    |   |     |    |     |    |    |         |    |    |
| 2  | 5.          |            | C.             | 1        | ha   |     |       | - 1   |     | 11  |      |     | C        |    |      | 2.1           | 2          | с.   | _    |    |   |     |    | 1   |    | 11 | • • • • | _  | ah |
| ۷. | 00          | 176        | TC             |          | ne   | 96  | iner  | r O I | SC  | JUI | 101  | ) 0 | T        |    | x )= | <u>3x</u>     | · Y .      | 110  | a    | me | P | art | cu | lar | SC |    | 100     | Su | cn |
|    | TING        | AT         | 710            | ))=      | - 2. |     |       |       |     |     |      |     |          |    |      |               |            |      |      |    |   |     |    |     |    |    |         |    |    |
|    | 4           | ¥          | 2              |          |      |     |       |       |     |     |      |     |          |    |      | 6             |            |      | -    |    |   |     |    |     |    |    |         |    |    |
|    | d           | <u>× =</u> | - 5            | x- 1     |      |     |       |       |     |     |      |     |          |    |      | e             | 1          | 0)=  | -2   |    |   |     |    |     |    |    |         |    |    |
|    | _1          | 1          |                | <b>.</b> |      |     |       |       |     |     |      |     |          |    |      |               | 0          |      |      |    |   |     |    |     |    |    |         |    |    |
|    | Y           | 0          | =              | 5×-      | ay   |     |       |       |     |     |      |     |          |    |      | _ <b>P</b> .( | ಲ್ -       | -Z   |      |    |   |     |    |     |    |    |         |    |    |
|    | 9           |            |                | C        |      |     |       |       |     |     |      |     |          |    |      |               |            |      |      |    |   |     |    |     |    |    |         |    |    |
|    | 74          | 70         | ۲ <sup>ع</sup> | 1 3 3    | 3ב   | dx  |       |       |     |     |      |     |          |    |      | A             | = - ;      | Z    |      |    |   |     |    |     |    |    |         |    |    |
|    |             |            |                |          | 1    | 5.  |       |       |     |     |      |     |          |    |      |               |            |      |      |    |   |     |    |     |    |    |         |    |    |
|    | In          | 41         | t c            | . =      | X    | 1+( | Z     |       |     |     |      |     |          |    |      |               |            |      |      |    |   |     |    |     |    |    |         |    |    |
|    |             |            |                | 2        |      |     |       |       |     |     |      |     |          |    |      |               |            |      |      | 3  |   |     |    |     |    |    |         |    |    |
|    | <u>In 1</u> | 41         | 11             | X        | +((  | 2-( | c,  ) |       |     |     |      |     |          |    |      | 46            | x)=        |      | Ze   | X  |   |     |    |     |    |    |         |    |    |
|    | 1.0         | 1.1        |                | 3        |      | (.) |       |       |     |     |      |     |          |    |      |               |            |      |      |    |   |     |    |     |    |    |         |    |    |
|    | e''         |            | = 6            | XŦ       | ננצי |     |       |       |     |     |      |     |          |    |      |               |            |      |      |    |   |     |    |     |    |    |         |    |    |
|    |             |            |                | e .      | 3    |     |       |       |     |     |      |     |          |    |      |               |            |      |      |    |   |     |    |     |    |    |         |    |    |
|    | ٦           | = (        | 22-            | • (      | e×_  |     |       |       |     |     |      |     |          |    |      |               |            |      |      |    |   |     |    |     |    |    |         |    |    |
|    |             |            |                |          | 2    |     |       |       |     |     |      |     |          |    |      |               |            |      |      |    |   |     |    |     |    |    |         |    |    |
|    | 41          | x) :       | = A            | ex.      |      |     |       |       |     |     |      |     |          |    |      |               |            |      |      |    |   |     |    |     |    |    |         |    |    |
|    |             |            |                |          |      |     |       |       |     |     |      |     |          |    |      |               |            |      |      |    |   |     |    |     |    |    |         |    |    |
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| New            | ron's   | L  | aw   | of   | С   | 00   | ino   | s  | late   | es ·   | tha   | + +   | he  | rat   | e (   | of (  | cool   | lina   | o  | Fa   | nc   | bj   | ect   | is   |   |  |   |
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| is 18          | 5°F   | an   | d  | is   | Pl  | aœ   | d   | on   | a  | tab  | le  | in  | a   | roo   | M   | wh  | ere  | #  | ne   | ter  | np   | era  | tur   | re   | is  | 75°1   | E.  |
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| New            | ron's   | j  | aw   | of   | C   | od   | ing   |  | =  | - K  | 4-  | To  | );  | Y   | (£):  | = +e  | emp  | . 0  | Fat  | jec  | + 0  | at t   | ime   | t  | >   |  | _   |
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| w ria          | TU  | 5  | Kn   | oω   | •   | 10 =   | +:  | ) r  | ,  | 1 (0   | )=  | כס  | r ,   | 40  | 50)   | >  ;  | 50-7   | •  |  |  |  |  |   |  |   |  | _   |
| Need           | t to  | £  | nd   | : K  |   | N  | (+)   |  |  | 5)   |   |   |   |   |   |   |  |  |  |  |  |  |   |  |   |  |   |
| NUCC           |   |  |  |  | • •   |  |   | ,  |  |  |   |   |   |   |   |   |  |  |  |  |  |  |   |  |   |  |   |
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| <u>dt</u> = -  | ·K (1   | -7   | 5)   |  |   |  |   |  |  |  |   |   | 0   | 710   | )=  | 185   | 5  |  |  | @  | Y(3  | 30):   | = 15  | 0  |   |  |   |
|                |   |  |  |  |   |  |   |  |  |  |   |   |   |   |   |   |  |  |  |  |  |  |   |  |   |  |   |
| y-79           | 5 dy  |  | K  | dt   |   |  |   |  |  |  |   |   | 18!   | 5=  | Ae  | +7  | <sup>2</sup> 5   |  |  | IIC  | e <sup>-</sup>   | 30 K   | +7  | 5 =  | 150   |  |   |
| 0              |   |  |  |  |   |  |   |  |  |  |   |   |   |   |   |   |  |  |  |  |  |  |   |  |   |  |   |
| 74-1           | F5 d  | =  | 5-1  | < dł   | ,   |  |   |  |  |  |   |   | 18  | 5 =   | Ał  | 75  | >  |  |  | 110  | e  | SOK.   | = 7   | 5  |   |  |   |
|                |   |  |  |  |   |  |   |  |  |  |   |   |   |   |   |   |  |  |  |  | 30K  |  | 76  |  |   |  |   |
| Inly           | -75   | =  | - K  | t †(   | C   |  |   |  |  |  |   |   | 110   | ) =   | A   |   |  |  |  | e_,  |  |  | +   | 110  |   |  |   |
|                | c   | 1  | (++  | с  |   |  |   |  |  |  |   |   |   |   |   |   |  |  |  | 21   |  |  | 1   | <u>s</u> ,   |   |  | _   |
| N-4            | 5 =   | e  |  |  |   |  |   |  |  |  |   |   |   |   |   |   |  |  |  | -3(  | )K=  | : Ir   |   | 0)   |   |  | _   |
|                | ck  | έL   | 70   |  |   |  |   |  |  |  |   |   |   |   |   |   |  |  |  | 10 -   |  |  |   | <u>75</u>  |   |  | _   |
| V = 6          | : e   | <b>-T</b>  | 73   | 2  |   |  |   |  |  |  |   |   |   |   |   |   |  |  |  | κ.=  |  | 50   | n   | 110  | )   |  | _   |
| M              | K   | t L  | 16   |  |   |  |   |  |  |  |   |   |   |   |   |   |  |  |  |  |  |  |   |  |   |  | _   |
| - <b>1</b> - P | ie<br>I   |  | TJ   |  |   |  |   |  |  |  |   |   |   |   |   |   |  |  |  |  |  |  |   |  |   |  | _   |
|                |   |  |  |  |   |  |   |  |  |  |   |   |   |   |   |   |  |  |  |  |  |  |   |  |   |  |   |
| SDec           | ifie  | d  | tir  | ne   | •   |  |   |  |  |  |   | Sir   | nD  | lif   | iec   | łł  | orr  | nu   | la   |  |  |  |   |  |   |  |   |
|                |   |  |  | 26   |   |  |   |  |  |  |   |   |   |   |   |   | 26   |  |  |  |  |  |   |  |   |  |   |
| ۲(f)=          | 110   | e <sup>30</sup>  | เทโ  | <u>110</u> )   | e<br>1  | 79   | 5   |  |  |  |   | 4   | t)=   | 110   | esi   | Inl   | tīō)(  | ÷ +  | 75   |  |  |  |   |  |   |  |   |
| -              |   |  |  | 75   | 111   |  |   |  |  |  |   |   |   |   |   |   | 15   | 1  |  |  |  |  |   |  |   |  |   |
| y(45)          | = 110   | )e <sup>30</sup>   | s in   | (110   | 1 -1  | +  | 75  | 5  |  |  |   |   | =   | 110   | elr   | il.   | ŽŽ)  |  | + 7  | 25   |  |  |   |  |   |  |   |
|                |   |  |  | -  |   |  |   |  |  |  |   |   |   |   |   |   |  |  |  |  |  |  |   |  |   |  |   |
|                |   |  | zin  | (10  | )   |  |   |  |  |  |   |   |   |   | 11  | 5   | 1/30   | ) <del> </del>   |  | _  |  |  |   |  |   |  |   |
|                | = 110   | e  | 210  | (110   | )   | +7   | 5   |  |  |  |   |   | :   | 110   | ( <u>1</u>  | <u>5</u> )  | 1/30   | ۲.   | - 7.   | 5  |  |  |   |  |   |  | _   |
|                | New<br>Prope<br>surra<br>is 18<br>Temp<br>Law<br>New<br>New<br>A<br>New<br>A<br>Need<br>OP<br>A<br>A<br>-7<br>S<br>-7<br>In In<br>N -7<br>S<br>-7<br>In In<br>N - 7<br>N = e<br>N = e<br>N = f<br>Spec<br>N(45) | Newton's<br>Proporti<br>surround<br>is 185°F<br>Tempera<br>Law of 0<br>Newton's<br>W hat w<br>Need to<br>Genera<br>$\frac{dy}{dt} = -k (y)$<br>$\frac{1}{y-75} dy$<br>$\frac{1}{y-75} dy$<br>$\frac{1}{y$ | Newton's La<br>proportiona<br>surrounding<br>is 185°F an<br>Temperatur<br>Law of Coo<br>Newton's La<br>What we<br>Need to fi<br>general f<br>$\frac{dy}{dt} = -k (y - 7)$<br>$\frac{1}{y - 75} dy = -$<br>$5 \frac{1}{y - 75} dy = -$<br>1 n 1y - 751 =<br>$y - 75 = e^{-1}$<br>$y - 75 = e^{-1}$<br>$y - 75 = e^{-1}$<br>$y = e^{c}e^{-kt} +$<br>$y = e^{c}e^{-kt} +$<br>$y = Ae^{-kt} +$ | Newton's Law<br>proportional<br>surroundings.<br>is 185°F and<br>Temperature<br>Law of Coolin<br>Newton's Law<br>What we Kn<br>Need to find<br>general for<br>$\frac{dy}{dt} = -K(y-75)$<br>$\frac{1}{y-75} dy = -K(y-75)$<br>$\frac{1}{y$ | Newton's Law of<br>proportional to<br>surroundings. A<br>is 185°F and is<br>Temperature of<br>Law of Cooling 1<br>Newton's Law of<br>What we know<br>Need to find: k<br>general formu<br>$\frac{dy}{dt} = -k (y - 75)$<br>$\frac{1}{y - 75} dy = -k dt$<br>$S \overline{y - 75} dy = -k dt$<br>$1 y - 75 = e^{-kt + c}$<br>$y - 75 = e^{-kt + c}$<br>$y = e^{c} e^{-kt} + 75$<br>$y = e^{c} e^{-kt} + 75$ | Newton's Law of C.<br>proportional to the<br>surroundings. A ro<br>is 185°F and is pl<br>Temperature of the<br>Law of Cooling to<br>Newton's Law of C<br>What we know: -<br>Need to find: k,<br>general formula<br>$\frac{dy}{dt} = -k (y - 75)$<br>$\frac{1}{y - 75} dy = -k dt$<br>S - k dt = S - k dt<br>$1 + 75 = e^{-kt + c}$<br>$y - 75 = e^{-kt + c}$<br>$y - 75 = e^{-kt + c}$<br>$y = e^{c}e^{-kt} + 75$<br>$y = e^{c}e^{-kt} + 75$<br>$y = Ae^{-kt} + 75$<br>$y = Ae^{-kt} + 75$ | Newton's Law of Cool<br>proportional to the t<br>surroundings. A roas<br>is 185°F and is place<br>Temperature of the<br>Law of Cooling to fir<br>Newton's Law of Cod<br>What we Know: To =<br>Need to find: K, Y<br>general formula:<br>$\frac{dy}{dt} = -k(y-75)$<br>$\frac{1}{y-75} dy = -k dt$<br>Sy-75 dy = S-k dt<br>1n  y-75  = -kt + c<br>$y = e^{c}e^{-kt} + 75$<br>$y = e^{c}e^{-kt} + 75$<br>$y = e^{c}e^{-kt} + 75$<br>$y = e^{c}e^{-kt} + 75$ | Newton's Law of Cooling<br>proportional to the tem<br>surroundings. A roast<br>is 185°F and is placed<br>Temperature of the two<br>Law of Cooling to Find<br>Newton's Law of Coding<br>What we Know: To = 75<br>Need to find: K, y(t)<br>general formula:<br>$\frac{dy}{dt} = -k(y-75)$<br>$\frac{1}{y-75}dy = -kdt$<br>$\frac{1}{y-75}dy = -kdt$<br>$\frac{1}{y-75}dy = -kdt$<br>$\frac{1}{y-75} = -kttc$<br>$y-75 = e^{-kt+c}$<br>$y = e^{e^{-kt}} + 75$<br>$y = Ae^{-kt} + 75$<br>$y = Ae^{-kt} + 75$<br>$y = 10e^{\frac{1}{30}\ln(\frac{75}{10})t} + 75$ | Newton's Law of Cooling si<br>proportional to the tempe<br>surroundings. A roast turl<br>is 185°F and is placed on<br>Temperature of the turke<br>Law of Cooling to Find the<br>Newton's Law of Coding: $k$<br>W hat we Know: To = 75°F<br>Need to find: K, Y(t),<br>general formula:<br>$\frac{dy}{dt} = -k(y-75)$<br>$\frac{1}{y-75} dy = -k dt$<br>1n 1y-751 = -k dt<br>1n 1y-751 = -k ttc<br>$y-75 = e^{-kt+c}$<br>$y = e^{c}e^{-kt} + 75$<br>$y = e^{c}e^{-kt} + 75$<br>$y = Ae^{-kt} + 75$<br>$y = Ae^{-kt} + 75$<br>$y = 10e^{\frac{1}{20} \ln(\frac{75}{10}) + 75} + 75$ | Newton's Law of Cooling state<br>proportional to the temperat<br>surroundings. A roast turkey<br>is 185°F and is placed on a<br>Temperature of the turkey f<br>Law of Cooling to find the turkey<br>Newton's Law of Coding: $y' = k = c$<br>What we know: To = 75°F, .<br>Need to find: k, y(t), y(t)<br>general formula:<br>$\frac{dy}{dt} = -k(y-75)$<br>$\frac{1}{y-75} dy = -k dt$<br>$S \frac{1}{y-75} dy = -k dt$<br>1 n 1y-751 = -k t + c<br>$y - 75 = e^{-kt + c}$<br>$y = e^{ce^{-kt}} + 75$<br>$y = Ae^{-kt} + 75$<br>$y(t) = 110e^{\frac{1}{30} \ln(\frac{10}{10}) + 5} + 75$ | Newton's Law of Cooling states<br>proportional to the temperature<br>surroundings. A roast turkey is<br>is 185°F and is placed on a tab<br>Temperature of the turkey falls<br>Law of Cooling to find the temp<br>Newton's Law of Coding: $y' = -K$<br>k = cooling<br>What we Know: To = 75°F, y lo<br>Need to find: $K$ , $y(t)$ , $y(45)$<br>general formula:<br>$\frac{dy}{dt} = -K(y-75)$<br>$\frac{1}{y-75} dy = -K dt$<br>$S \frac{1}{y-75} dy = -K dt$<br>1n   y-75  = -K t + C<br>$y = e^{c} e^{kt} + 75$<br>$y = A e^{-Kt} + 75$<br>$y(t) = 110 e^{\frac{1}{30} \ln(\frac{110}{10}) + \frac{1}{75} + 75}$ | Newton's Law of Cooling states that<br>proportional to the temperature d<br>surroundings. A roast turkey is taken<br>is 185°F and is placed on a table<br>Temperature of the turkey falls to<br>Law of Cooling to find the temper<br>Newton's Law of Cooling: $y' = -K(y)$<br>k = cooling<br>What we Know: To = 75°F, $y(0) = 1$<br>Need to find: $K$ , $y(t)$ , $y(145)$<br>general formula:<br>$\frac{dy}{dt} = -K(y-75)$<br>$\frac{1}{y-75}dy = -Kdt$<br>1n  y-75] = -Kdt<br>1n  y-75] = -Kt + c<br>$y = e^{c}e^{-Kt} + 75$<br>$y = Ae^{-Kt} + 75$<br>$y(t) = 110e^{\frac{1}{10}\ln[\frac{75}{10}] + 75}$<br>$y(145) = 110e^{\frac{1}{20}\ln[\frac{75}{10}] + 75}$ | Newton's Law of Cooling states that the proportional to the temperature diffusion of the temperature diffusion of the second of the turkey is taken is 185°F and is placed on a table in Temperature of the turkey falls to 1 Law of Cooling to find the temperature of the turkey is taken to find the temperature of the turkey to 1 Law of Cooling to find the temperature of the turkey is taken to 1 Law of Cooling to 1 Law of Cool | Newton's Law of Cooling states that the<br>proportional to the temperature difference<br>surroundings. A roast turkey is taken finits<br>185°F and is placed on a table in a representative of the turkey falls to 150°<br>Law of Cooling to find the temperature<br>Newton's Law of Coding: $v' = -K (v - T_0)$ ;<br>$k = cooling construction What we know: T_0 = 75°F, v(0) = 185°F,Need to find: k, v(t), v(45)general formula:\frac{1}{V-75} dv = -K dt1 - 75 = e^{-Kt+c}v = e^{c}e^{-Kt+c}1 - 8e^{-Kt} + 75v = Ae^{-Kt} + 75v(t) = 110e^{\frac{1}{20} \ln(\frac{15}{110})t} + 75v(t) = 110e^{\frac{1}{20} \ln(\frac{75}{110})t} + 75v = 10e^{\frac{1}{20} \ln(\frac{75}{110})t} + 75v = 10e^{\frac{1}{20} \ln(\frac{75}{110})t} + 75v = 10e^{\frac{1}{20} \ln(\frac{75}{110})t} + 75v = 75$ | Newton's Law of Cooling states that the rate<br>proportional to the temperature difference<br>surroundings. A roast turkey is taken from<br>is 185°F and is placed on a table in a room<br>Temperature of the turkey falls to 150°F a<br>Law of Cooling to find the temperature of<br>Newton's Law of Coding: $v' = -K (v - T_0)$ ; $v_k$<br>k= cooling constant<br>What we know: $T_0 = 75°F$ , $v_1(0) = 185°F$ , $v_1(0)$<br>Need to find: k, $v_1(t)$ , $v_1(u_5)$<br>general formula:<br>$\frac{1}{v_1 - 75} dv = -K dt$<br>$\frac{1}{v_1 - 75} dv = -K dt$<br>10 = 185 =<br>$S \frac{1}{v_1 - 75} dv = -K dt$<br>$10 = 10 = \frac{-K t + c}{2}$<br>$10 = 10 = \frac{-K t + c}{2}$<br>$10 = 10 = \frac{1}{20} \ln(\frac{75}{10}) + 75$<br>$10 = 10 = \frac{1}{20} \ln(\frac{75}{10}) + 10 = $ | Newton's Law of Cooling states that the rate of<br>proportional to the temperature difference be<br>surroundings. A roast turkey is taken from a<br>is 185°F and is placed on a table in a room<br>Temperature of the turkey falls to 150°F afte<br>Law of Cooling to find the temperature of the<br>Newton's Law of Coding: $y' = -K (y - T_0)$ ; $y(t)$ .<br>K = cooling constant, T<br>What we know: $T_0 = 75°F$ , $y(0) = 185°F$ , $y(30)$<br>Need to find: $K$ , $y(t)$ , $y(45)$<br>general formula:<br>y' = -K (y - 75)<br>y' = -K dt<br>y' = 75 dy = -K dt<br>y' = 75 dy = -K dt<br>$y = e^c e^{-Kt} + 75$<br>$y = e^{-Kt + c}$<br>$y = e^c e^{-Kt} + 75$<br>$y' = A e^{-Kt} + 75$<br>$y' = 110 e^{\frac{1}{25} \ln(\frac{150}{10})^{45}} + 75$<br>$y' = 110 e^{\frac{1}{25}}$ | Newton's Law of Cooling states that the rate of operational to the temperature difference between the surroundings. A roast turkey is taken from an is 185°F and is placed on a table in a room when temperature of the turkey falls to 150°F after 1 Law of Cooling to find the temperature of the turkey falls to 150°F after 1 Law of Cooling to find the temperature of the turkey falls to 150°F after 1 Law of Cooling to find the temperature of the turkey falls to 150°F after 1 Law of Cooling to find the temperature of the turkey falls to 150°F after 1 Law of Cooling to find the temperature of the turkey falls to 150°F after 1 Law of Cooling to find the temperature of the turkey falls to 150°F after 1 Law of Cooling to find the temperature of the turkey falls to 150°F after 1 Law of Cooling to find the temperature of the turkey falls to 150°F after 1 Law of Cooling to find the temperature of the turkey falls to 150°F after 1 Law of Cooling to find the temperature of the turkey falls to 150°F after 1 Law of Cooling to find the temperature of the turkey falls to 150°F after 1 Law of Cooling to find the temperature of the turkey falls to 150°F after 1 Law of Cooling to find the temperature of the turkey falls to 150°F after 1 Law of Cooling to find the temperature of the turkey falls to 150°F after 1 Law of Cooling to find the temperature of the two cooling constant, To= $Newton's Law of Cooling to find the temperature of the two cooling constant to the temperature of the two cooling constant, to= Newton's Law of Cooling to find the temperature of the two cooling constant, to= Newton's Law of Cooling to find the temperature of the two cooling constant, to= Newton's Law of Cooling to find the temperature of the temperature of the two cooling constant, to= Newton's Law of Cooling to find the temperature of the temp$ | Newton's Law of Cooling states that the rate of cool<br>proportional to the temperature difference between<br>surroundings. A roast turkey is taken from an over<br>is 186°F and is placed on a table in a room where<br>Temperature of the turkey falls to 150°F after half<br>Law of Cooling to find the temperature of the tur<br>Newton's Law of Coding: $Y = -K (Y - T_0)$ ; $Y(E) = tempK = cooling constant, T_0 = amlWhat we know: T_0 = 75°F, Y(0) = 185°F, Y(30) = 150°FNeed to find: K, Y(E), Y(ES)general formula:\frac{1}{Y-75} dy = -K dE\frac{1}{Y-75} dy = -K dE185 = Ae^0 + 75\frac{1}{Y-75} dy = S-K dE10 = AY-75 = e^{-KE+C}Y = e^{e^{-KE}} + 75Y = Ae^{-KE} + 75Y = 100 e^{\frac{1}{20} \ln(\frac{10}{10}) + 5} + 75$ | Newton's Law of Cooling states that the rate of cooling<br>proportional to the temperature difference between<br>surroundings. A roast turkey is taken from an oven i<br>is 185°F and is placed on a table in a room where the<br>Temperature of the turkey falls to 150°F after half a<br>Law of Cooling to find the temperature of the turker<br>Newton's Law of Coding: $V = -K (V - T_0)$ ; $V(t) = temp. ofK = cooling constant, T_0 = ambienWhat we know: T_0 = 75°F, V(0) = 185°F, V(30) = 150°FNeed to find: K, V(t), V(45)general formula:V = -K (V - 75)V = -K (V - 75)$ | Newton's Law of Cooling states that the rate of cooling of<br>proportional to the temperature difference between the<br>surroundings. A roast turkey is taken from an oven when<br>is 185°F and is placed on a table in a room where the<br>Temperature of the turkey falls to 150°F after half an N<br>Law of Cooling to find the temperature of the turkey of<br>Newton's Law of Coding: $\sqrt{=}-K(1,-T_0)$ ; $\sqrt{10}=1emp$ . of ot<br>K= cooling constant, To= ambient<br>What we Know: To=75°F, $\sqrt{10}=185°F$ , $\sqrt{130}=150°F$<br>Need to find: K, $\sqrt{11}$ , $\sqrt{145}$<br>general formula:<br>$\sqrt{1}+75 dy = -K dt$<br>$\sqrt{-75} dy = -K dt$<br>$\sqrt{-75} dy = S-K dt$<br>$\sqrt{-75} dy = S-K dt$<br>$\sqrt{-75} = e^{-Kt+c}$<br>$\sqrt{-75} = e^{-Kt$ | Newton's Law of Cooling states that the rate of cooling of a proportional to the temperature difference between the c<br>surroundings. A roast turkey is taken from an oven when is 185°F and is placed on a table in a room where the temperature of the turkey falls to 150°F after half an hou Law of Cooling to find the temperature of the turkey after half an hou Law of Cooling to find the temperature of the turkey after half an hou Law of Cooling to find the temperature of the turkey after half an hou Law of Cooling to find the temperature of the turkey after half an hou Law of Cooling to find the temperature of the turkey after half an hou Law of Cooling to find the temperature of the turkey after half an hou Law of Cooling to find the temperature of the turkey after half an hou Law of Cooling to find the temperature of the turkey after half an hou Law of Cooling to find the temperature of the turkey after half an hou Law of Cooling to find the temperature of the turkey after half an hou Law of Cooling to find the temperature of the turkey after half an hou Law of Cooling to find the temperature of the turkey after half an hou Law of Cooling to find the temperature of the turkey after half an hou Law of Cooling to find the temperature of the turkey after half an hou Law of Cooling to find the temperature of the turkey after half an hou Law of Cooling to find the temperature of the turkey after half an hou Law of Cooling to find the temperature of the turkey after half an hou Law of Cooling to find the temperature of the turkey after half an hou taw of Cooling to find the temperature of the turkey after half an hou taw of Cooling to find the temperature of the turkey after half an hou taw of cooling constant, To a mobient ter half an hou taw of the turkey after half an hou taw of the taw of the turkey after half an hou taw of the taw of the | Newton's Law of Cooling states that the rate of cooling of an operational to the temperature difference between the obj<br>surroundings. A roast turkey is taken from an oven when its<br>is 185°F and is placed on a table in a room where the temp<br>Temperature of the turkey falls to 150°F after half an hour.<br>Law of Cooling to find the temperature of the turkey after<br>Newton's Law of Coding: $v' = -K (v - T_0)$ ; $v(e) = temp. of object ofK = cooling constant, T_0 = ambient tempWhat we know: T_0 = 75°F, v(o) = 185°F, v(so) = 150°FNeed to find: K, v(e), v(us)v(a) = 185°F, v(so) = 160°FNeed to find: K, v(e), v(us)v(b) = 185°F, v(so) = 185°Fv(b) = 185°Fv(c) = 10°Fv(c) = 10°F$ | Newton's Law of Cooling states that the rate of cooling of an obj<br>proportional to the temperature difference between the object<br>surroundings. A roast turkey is taken from an oven when its the<br>is 185°F and is placed on a table in a room where the temperature<br>Temperature of the turkey falls to 150°F after half an hour. Ap<br>Law of Cooling to find the temperature of the turkey after 45<br>Newton's Law of Coding: $v' = -K (v - T_0)$ ; $v(t) = temp. of object at the k = cooling constant, T_0 = ambient temp.What we Know: T_0 = 75°F, v(0) = 185°F, v(30) = 150°FNeed to find: K, v(t), v(45)general formula:v - 75 dv = -K dt10 e^{30k}v - 75 dv = -K dt10 e^{30k}10 e^{30k}10 e^{30k}10 e^{30k}10 e^{30k}10 e^{30k}10 e^{30k} = -K dt10 e^{30k} = 10 e^{-8k+c}10 e^{-30k} = 10 e^{-8k+c}10 e^{-8k+c}$ | Newton's Law of Cooling states that the rate of cooling of an object as<br>proportional to the temperature difference between the object as<br>surroundings. A roast turkey is taken from an oven when its temp<br>is 185° f and is placed on a table in a room where the temperatur<br>Temperature of the turkey falls to 150° f after half an hour. Appl-<br>Law of Cooling to find the temperature of the turkey after 45 m<br>Newton's Law of Coding: $v_1 = -K(v_1 - T_6)$ ; $v_1(s) = temp. of object at time K = cooling constant, T_6 = ambient temp.What we know: T_6 = 75^6 F, v_1(0) = 185° F, v_1(30) = 150° FNeed to find: K, v_1(t), v_1(45)general formula:v_1 - 75 dv = -K dtv_1 - 75$ | Newton's Law of Cooling states that the rate of cooling of an object is<br>proportional to the temperature difference between the object and<br>surroundings. A roast turkey is taken from an oven when its tempera-<br>is 186° F and is placed on a table in a room where the temperature.<br>Temperature of the turkey falls to 150° F after half an hour. Apply T<br>Law of Cooling to find the temperature of the turkey after 45 minu<br>Newton's Law of Coding: $V = -K (V - T_0)$ ; $V(E) = temp. of object at time t K = cooling constant, T_0 = ambient temp.What we know: T_0 = 75°F, V(0) = 185°F, V(30) = 150°FNeed to find: K, V(E), V(45)general formula:V = -K (V - 75)V = -K (V - 75)$ | Newton's Law of Cooling states that the rate of cooling of an object is<br>proportional to the temperature difference between the object and its<br>surroundings. A roast turkey is taken from an oven when its temperature is<br>Temperature of the turkey falls to 150°F after half an hour. Apply New<br>Law of Cooling to find the temperature of the turkey after '15 minutes<br>Newton's Law of Coding: $v = -K (v - T_0)$ ; $v (v) = temp. of object at time t$ ,<br>$k = cooling constant, T_0 = 75°F$ , $v (0) = 185°F$ , $v(30) = 150°F$<br>Need to find: $k$ , $v(t)$ , $v(45)$<br>general formula:<br>$\frac{dv}{dt} = -K (v - 75)$<br>$\frac{1}{v - 75} dv = -K dt$<br>$10 e^{-30k} + 75 = 150$<br>$\frac{1}{v - 75} dv = -K dt$<br>$10 e^{-30k} = 75^{\circ}F$<br>$10 e^{-30k} = 75^$ | Newton's Law of Cooling states that the rate of cooling of an abject is<br>proportional to the temperature difference between the object and its<br>surroundings. A reast turkey is taken from an oven when its temperature<br>is 185° and is placed on a table in a room where the temperature is 75°<br>Temperature of the turkey falls to 150° f after half an hour. Apply Newton<br>Law of Cooling to find the temperature of the turkey after '15 minutes.<br>Newton's Law of Coding: $\sqrt{2} = -K(1-T_0)$ ; $\gamma(10) = 100° f$<br>Newton's Law of Coding: $\sqrt{2} = -K(1-T_0)$ ; $\gamma(10) = 150° f$<br>Newton's Law of Coding: $\sqrt{2} = -K(1-T_0)$ ; $\gamma(10) = 150° f$<br>Need to find: $K$ , $\gamma(10)$ , $\gamma(145)$<br>general formula:<br>$\sqrt{1}$ initial Conditions:<br>$\sqrt{4}$<br>$\sqrt{4}$ is $\sqrt{1}$ initial Conditions:<br>$\sqrt{1}$ initial Conditions:<br>$\sqrt{1}$ is $\sqrt{1}$ is $\sqrt{1}$ is $\sqrt{1}$ initial C |

## **Exit Ticket** Polar Integrals

**Polar Integrals** The integral over region  $D = \{(r, \theta) \mid a \le r \le b, c \le \theta \le d\}$  is

$$\int \int_D f(r,\theta) \, dA = \int_c^d \int_a^b f(r,\theta) \cdot r \cdot dr d\theta$$

Find the integral  $\int \int_D 2x^2 + y^2 \, dA$  for the regions bounded by the following:

1. the circles of radius 1 and 3 centered at the origin 
$$\int_{0}^{2\pi} \int_{0}^{1} r^{2} \cos^{2}\theta + r^{3} r dr d\theta$$
  
 $= \int_{0}^{2\pi} \int_{0}^{1} r^{2} \cos^{2}\theta + r^{3} r dr d\theta$   
 $= \int_{0}^{2\pi} \int_{0}^{1} r^{2} \cos^{2}\theta + r^{3} r dr d\theta$   
 $= \int_{0}^{2\pi} \int_{0}^{1} r^{2} \cos^{2}\theta + r^{3} r dr d\theta$   
 $= \int_{0}^{2\pi} \int_{0}^{1} r^{2} \cos^{2}\theta + r^{3} r dr d\theta$   
 $= \int_{0}^{2\pi} \int_{0}^{1} r^{2} \cos^{2}\theta + r^{3} r dr d\theta$   
 $= \int_{0}^{2\pi} \int_{0}^{1} r^{2} \cos^{2}\theta + r^{3} r dr d\theta$   
 $= \int_{0}^{2\pi} \int_{0}^{1} r^{2} \cos^{2}\theta + r^{3} r dr d\theta$   
 $= \int_{0}^{2\pi} \int_{0}^{1} r^{2} (\cos^{2}\theta + r) \int_{0}^{1} d\theta$   
 $= \int_{0}^{2\pi} \int_{0}^{1} (r^{2} \cos^{2}\theta + r) \int_{0}^{1} d\theta$   
 $= \int_{0}^{2\pi} \int_{0}^{1} (r^{2} \cos^{2}\theta + r) \int_{0}^{1} d\theta$   
 $= \int_{0}^{2\pi} \int_{0}^{1} (r^{2} \cos^{2}\theta + r) \int_{0}^{1} d\theta$   
 $= \int_{0}^{2\pi} \int_{0}^{1} (r^{2} \cos^{2}\theta + r) \int_{0}^{1} d\theta$   
 $= \int_{0}^{2\pi} \int_{0}^{1} (r^{2} \cos^{2}\theta + r) \int_{0}^{1} d\theta$   
 $= \int_{0}^{2\pi} \int_{0}^{1} (r^{2} \cos^{2}\theta + r) \int_{0}^{1} d\theta$   
 $= \int_{0}^{2\pi} \int_{0}^{1} (r^{2} \cos^{2}\theta + r^{2}) \cdot r dr d\theta$   
 $= \int_{0}^{\pi} \int_{0}^{1} (r^{2} \cos^{2}\theta + r^{2}) \cdot r dr d\theta$   
 $= \int_{0}^{\pi} \int_{0}^{1} (r^{2} \cos^{2}\theta + r) d\theta$   
 $= \int_{0}^{\pi} \int_{0}^{1} (r^{2} \cos^{2}\theta + r) d\theta$   
 $= \int_{0}^{\pi} \int_{0}^{\pi} (r^{2} (\cos^{2}\theta + r)) d\theta$   
 $= \int_{0}^{\pi} \int_{0}^{\pi} (\cos^{2}\theta + r) d\theta$   

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