Integrations

1–64

· 18, 17, 21, 25, 32, 58, 43, 7, 3, 4

18. $\int_4^9 \frac{dt}{(t^2-1)^2}$ Partial Fraction

$\int_4^9 \frac{1}{(t^2-1)^2}\,dt$ *Check for improper integrals

$t^2-1=0$

$t^2=1$

$\frac{1}{(t^2-1)^2} = \frac{Ax+B}{(t^2-1)} + \frac{Cx+D}{(t^2-1)^2}$ $t=\pm 1 \Rightarrow$ not in range

↳ This puts you right back to where you started

Factor	Degree (A or Ax+b)	Multiplicity
(t^2-1)	2	2

$\int_4^9 \frac{1}{(t+1)^2(t-1)^2}\,dt$

Factor	Degree	Multiplicity
$(t+1)$	1	2
$(t-1)$	1	2

$\frac{1}{(t+1)(t-1)} = \frac{A}{t+1} + \frac{B}{(t+1)^2} + \frac{C}{(t-1)} + \frac{D}{(t-1)^2}$ NOT POSSIBLE FOR TEST BC

$1 = A(t+1)(t-1)^2 + B(t-1)^2 + C(t-1)(t+1)^2 + D(t+1)^2$ TOO LONG

17. $\int \frac{6x+4}{x^2-1}\,dx$ *always test for U-sub 1st

$\int \frac{6x}{x^2-1} + \frac{4}{x^2-1}\,dx$ easier to solve partial fractions w/ constant on top

$u=x^2-1$

$3\ln|x^2-1| + \int \frac{4}{(x+1)(x-1)}\,dx$

$\frac{4}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$ $x=1$ $x=-1$

$4=A(x-1)+B(x+1)$

$4=2B$ $4=-2A$

$B=2$ $A=-2$

$3\ln|x^2-1| + \int \frac{-2}{x+1} + \frac{2}{x-1}\,dx$

$3\ln|x^2-1| - 2\ln|x+1| + 2\ln|x-1| + C$

ANOTHER WAY FOR PARTIAL FRACTIONS:

$4=Ax-A+Bx+B$

$x:\ 0x=Ax+Bx$

$0=A+B$

$C:\ 4=-A+B$

$4+A=B$

$0=A+4+A$

$-4=2A$ $A=-2$

21. $\int_0^1 \ln(4-2x)\,dx$

$4-2x=0$
$4=2x$
$2=x$

$u=\ln(4-2x)$
$du=\frac{1}{4-2x}\cdot -2$
$=\frac{-1}{2-x}=\frac{1}{x-2}$

$dv=dx$
$v=x$

$=\ln(4-2x)x-\int x\cdot\frac{1}{x-2}\,dx$

$u=x-2$
$x=u+2$
$du=dx$

$\ln(4-2x)\cdot x-\int\frac{u+2}{u}\,du$

$\ln(4-2x)\cdot x-\int\frac{u}{u}+\frac{2}{u}\,du$

$x\ln(4-2x)-u+2\ln|u|+C$

$\ln(4-2x)\cdot x-(x-2)+2\ln|(x-2)|+C$

25. $\int_6^{\frac{\pi}{4}} \sin(\overset{A}{3x})\cos(\overset{B}{5x})\,dx$

$\sin A\cos B = \frac{1}{2}[\sin(A+B)+\sin(A-B)]$

$\int_0^{\frac{\pi}{4}} \frac{1}{2}[\sin(3x+5x)+\sin(3x-5x)]\,dx$

$\frac{1}{2}\int_0^{\frac{\pi}{4}} \sin(8x)+\sin(-2x)\,dx$

$\frac{1}{2}\left[-\cos 8x\cdot\frac{1}{8} - \cos(-2x)\cdot\frac{-1}{2}\right]_0^{\frac{\pi}{4}}$

$\frac{-1}{16}\cos 8x + \frac{1}{4}\cos(-2x)\Big]_0^{\frac{\pi}{4}}$ then plug in

32. $\int_{\frac{\pi}{2}}^{\pi} \cot^2\left(\frac{\theta}{2}\right) d\theta$

$1 = \cos^2 + \sin^2$

$\csc^2 = \cot^2 + 1$

$\cot^2 = \csc^2 - 1$

$\int_{\frac{\pi}{2}}^{\pi} \csc^2\left(\frac{\theta}{2}\right) - 1 \, d\theta$

$\frac{d}{dx}\left[\cot(x)\right] = -\csc^2 x$

$\left. -\cot\left(\frac{\theta}{2}\right) \cdot 2 - \theta \right]_{\frac{\pi}{2}}^{\pi}$

DON'T THINK PROBLEM WLD BE GIVEN BC NOT FAMILIAR

can check by doing derivative

can be expected as short response to use things you don't think about

58. $\int \sin(x)\cosh(x)\,dx$

hyperbolic

won't be dealing with ∴ not possible for us to do

43. $\int \frac{16}{(x-2)^2(x^2+4)}\,dx$

could be set up but do not solve for test

Factor	Deg	Mult
$(x-2)$	1	2
(x^2+4)	2	1

$$\int \frac{A}{x-2} + \frac{B}{x-2^2} + \frac{Cx+D}{x^2+4}\,dx$$

ROOTS method fails

7. $\int \frac{1}{x(x^2-1)^{3/2}}\,dx$

$\int$

$u = x^2-1$
$du = 2x\,dx$
$\frac{1}{2x}du = dx$
$(u-1)^{1/2} = x$

*Partial Fractions only works w/ whole numbers powers

$x = \sec\theta$
$dx = \tan\theta\sec\theta\,d\theta$

$$\int \frac{1}{\sec\theta(\sec^2\theta-1)^{3/2}} \cdot \tan\theta\sec\theta\,d\theta$$

$$\int \frac{1}{\sec\theta(\tan^2\theta)^{3/2}} \cdot \tan\theta\sec\theta\,d\theta$$

$$\int \frac{\cancel{\tan\theta\sec\theta}}{\cancel{\sec\theta}\cdot\tan^{3}\theta^{2}}\,d\theta$$

$$\int \cot^2\theta\,d\theta = \int \csc^2\theta - 1\,d\theta$$

could be given w/ take x to be sec

3. $\int \cos^3\theta \sin^8\theta \, d\theta$ trig sub

$\int \cos\theta \cos^2\theta \cdot \sin^8\theta \, d\theta$

$\int \cos\theta \cdot (1-\sin^2\theta) \sin^8\theta \, d\theta$ $\quad u = \sin\theta$

$du = \cos\theta \, d\theta$

$\int \cos\theta \cdot (\sin^8\theta - \sin^{10}\theta) d\theta$

$\int u^8 - u^{10} du$

$\frac{u^9}{9} - \frac{u^{11}}{11} + C = \frac{(\sin^9\theta)}{9} - \frac{(\sin^{11}\theta)}{11} + C$

At most would see $(\sin^2\theta)(\cos^2\theta)$ for even powers

probably MC problem

4. $\int x e^{-12x} dx$ Integration by parts

$u = x \qquad dv = e^{-12x}$

$du = dx \qquad v = -\frac{1}{12} e^{-12x}$

$uv - \int v \, du$

$(x)\left(-\frac{1}{12} e^{-12x}\right) - \int -\frac{1}{12} e^{-12x} dx$

$-\frac{x}{12} e^{-12x} + \frac{1}{12} \int e^{-12x} dx$ $\quad$ if definite integral plug in at the end

$-\frac{x}{12} e^{-12x} - \frac{1}{144} e^{-12x} + C$