Estimating Partial Derivatives

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Estimating	One Varible	Two Variables (y)	Two Variable (x)
Forward Difference	$f'(x) = \frac{f(x+h)-f(x)}{h}$	$f_y = \frac{f(x,y+h)-f(x,y)}{h}$	$f_x = \frac{f(x+h,y)+f(x,y)}{h}$
Backward Difference	$f'(x) = \frac{f(x)-f(x-h)}{h}$	$f_y = \frac{f(x,y)-f(x,y-h)}{h}$	$f_x = \frac{f(x,y)-f(x-h,y)}{h}$
Central Difference	$f'(x) = \frac{f(x+h)-f(x-h)}{2h}$	$f_y = \frac{f(x,y+h)-f(x,y-h)}{2h}$	$f_x = \frac{f(x+h,y)-f(x-h,y)}{2h}$

Example:

1. A two-variable function $f(x,y)$ has selected values given by

y\x	2.5	3.0	3.5
-1.0	6.0	6.5	8.0
-1.5	6.5	7.0	8.5
-2.0	5.8	6.9	7.8

(a) Write down three estimates for the value of $\frac{\partial f}{\partial y}(3.5,-1.5)$. State what estimates they are.

Central difference: $\frac{1}{\Delta y}[f(x,y+h)-f(x,y-h)]$

$$\frac{\partial f}{\partial y}(3.5,-1.5) = \frac{1}{\Delta y}[f(3.5,-1.0) - f(3.5,-2.0)] \quad ; \quad \Delta y = -1-(-2) = 1$$
$$= 1[8-7.8]$$
$$= 1[0.2]$$
$$= 0.2$$

Forward difference: $\frac{1}{\Delta y}[f(x,y+h)-f(x,y)]$

$$\frac{\partial f}{\partial y}(3.5,-1.5) = \frac{1}{\Delta y}[f(3.5,-1.0) - f(3.5,-1.5)] \quad ; \quad \Delta y = -1-(-1.5) = 0.5$$
$$= \frac{1}{0.5}[8-8.5]$$
$$= 2[-0.5]$$
$$= -1$$

Backward difference: $\frac{1}{\Delta y}[f(x,y)-f(x,y-h)]$

$$\frac{\partial f}{\partial y}(3.5,-1.5) = \frac{1}{\Delta y}[f(3.5,-1.5) - f(3.5,-2.0)] \quad ; \quad \Delta y = -1.5-(-2) = 0.5$$
$$= \frac{1}{0.5}[8.5-7.8]$$
$$= 2[0.7]$$
$$= 1.4$$

(b) Estimate $\frac{\partial f}{\partial x}(3.5,-1.5)$. Which did you use?

We can only look backwards:

$$\frac{\partial f}{\partial x}(3.5,-1.5) = \frac{1}{\Delta x}\left[f(3.5,-1.5) - f(3.0,-1.5)\right] \quad ; \quad \Delta x = 3.5-3 = 0.5$$

$$= \frac{1}{0.5}[8.5-7]$$

$$= 2[1.5]$$

$$= 3$$

Part b points out a limitation on each of these estimations, they require multiple data points and data tables are often not infinite. Each corner & edge will be limited in what estimate you can do.

y \ x	x_1	x_2	x_3
y_1			
y_2			
y_3			

* note: if you can't do forward & backwards you can't do central

top line: unable to do backward y (needs y_0)

bottom line: unable to do forward y (needs y_4)

right side: unable to do forward x (needs x_4)

left side: unable to do backward x (needs x_0)

top left corner: unable to use backward y and backward x

top right corner: unable to use backward y and forward x

bottom left corner: unable to do forward y and backward x

bottom right corner: unable to do forward y and forward x

Exit Ticket Partial Derivatives

Partial Derivatives A derivative asks how much does y "moves" when we vary x. Partial derivatives are the multi-variable version of this process.

- $\dfrac{\partial}{\partial x}[f(x,y)] = f_x =$ take the derivative with respect to x while keeping y constant
- $\dfrac{\partial}{\partial y}[f(x,y)] = f_y =$ take the derivative with respect to y while keeping x constant

Find all four partial second derivatives f_{xx}, f_{yy}, f_{xy}, f_{yx}:

1. $8xe^{6x-y^2}$

$f_x = 8e^{6x-y^2} + 48xe^{6x-y^2}$

$f_{xx} = 48e^{6x-y^2} + 48e^{6x-y^2} + 288xe^{6x-y^2}$

$f_y = -2xye^{6x-y^2}$

$f_{yy} = -2xe^{6x-y^2} + 4xye^{6x-y^2}$

$f_{xy} = f_{yx} = -16y^2e^{6x-y^2} - 96xye^{6x-y^2}$

2. $\ln(5x^2 - y)$

$f_x = \dfrac{10x}{5x^2-y}$

$f_{xx} = \dfrac{10(5x^2-y) - 10x(10x)}{(5x^2-y)^2}$

$f_y = \dfrac{-1}{5x^2-y} = -(5x^2-y)^{-1}$

$f_{yy} = -(5x^2-y)^{-2}$

$f_{xy} = f_{yx} = (5x^2-y)^{-2} \cdot 10x$

3. $\cos(3x)y^2$

$f_x = -3\sin(3x)y^2$

$f_{xx} = -9\cos(3x)y^2$

$f_y = 2\cos(3x)y$

$f_{yy} = 2\cos(3x)$

$f_{xy} = -6\sin(3x)y$

4. $\dfrac{x-1-2y}{x^2} = \dfrac{x}{x^2} - \dfrac{1}{x^2} - \dfrac{2y}{x^2}$

$= x^{-1} - x^{-2} - 2yx^{-2}$

$f_x = -x^{-2} + 2x^{-3} + 4yx^{-3}$

$f_{xx} = 2x^{-3} - 6x^{-4} - 12yx^{-4}$

$f_y = -2x^{-2}$

$f_{yy} = 0$

$f_{xy} = f_{yx} = 4x^{-3}$