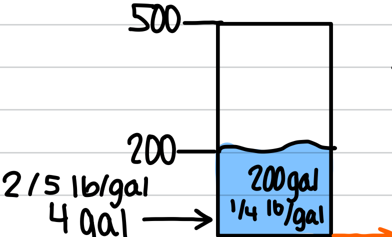
# Review 4/16/25
## Tank Problems

1. A 500 gallon tank contains 200 gallons of brine w/ concentration 1/4 lbs/gal. Brine containing 2/5 lb/gal is pumped into the tank at a rate of 4 gal/min

a) If $y(t)$ is the amount of salt in the tank at time $t$, find the diff eq.




rate out

amount of salt changing per minute

$$\frac{dy}{dt} = \text{salt in} - \text{salt out}$$

lb/min · min

$$= \text{conc}\cdot(\text{rate in}) - \text{con}\cdot(\text{rate out})$$

$$= \frac{2}{5}(4) - \left(\frac{y(t)}{V(t}\right)0$$

$$* \text{con out} = \frac{y(t)}{V(t)} = \frac{y(t)}{200+4t-0t} \qquad \int \frac{dy}{dt} = \int \frac{8}{5} = \frac{8}{5}t$$

ex) if rate out = 2 then $= \dfrac{y(t)}{200-4t-2t}$

b) When does the tank overflow

$V(t) = 500$

$200 + 4t - 0t = 500$

$4t = 300$

$t = \dfrac{300}{4} = \dfrac{3(100)}{4} = 75$

$$\frac{dy}{dt} = \frac{8}{5} - \frac{y(t)}{200+4t}(0)$$

$$\int\frac{dy}{dt} = \int\frac{8}{5}$$

$$y = \frac{8}{5}t + C$$

Replace w/ rate out $= 2$

1st order diff eq.
$y' + A(x)y = B(x)$

$$\frac{dy}{dt} = \frac{8}{5} - \frac{y(t)}{200+2t}(2)$$

$$y' + \frac{1}{100+t}y = \frac{8}{5}$$

Step 1: $A(x) = \frac{1}{100+t}$

$B(x) = \frac{8}{5}$

$$\alpha = e^{\int\frac{1}{100+t}dt}$$

$$\alpha = e^{\ln|100+t|}$$

$$= 100+t$$

Step 2: find $\alpha = e^{\int A(x)dx}$

Step 3: $y(x) = \frac{1}{\alpha(x)}\left[\int \alpha(x)B(x)dx + C\right]$

$$y(x) = \frac{1}{100+t}\left[\int(100+t)\left(\frac{8}{5}\right)dt + C\right]$$

$$y(x) = \frac{1}{100+t}\left[\int\frac{800}{5} + \frac{8}{5}t\,dt + C\right]$$

$$y(x) = \frac{1}{100+t}\left[\frac{800}{5}t + \frac{8}{10}t^2 + C\right]$$

$$y(x) = \frac{1}{100+t}\left[\frac{800}{5}t + \frac{4}{5}t^2 + C\right]$$

inital value
$y(0) = C(0) \cdot V(0)$
$= \frac{1}{4}(200)$
$= 50$

## Patient Problem Geometric Series

2. A patient is injected once a day w/ 90 units of a drug. Suppose the drug is eliminated w/ any single injection leaving an amount of $90e^{-.4t}$ remaining after t days.

a) what is the amt of drug in the body 1 day later?

$$90e^{-.4(1)} = 90e^{-.4}$$

b) 2 days

$$90e^{-.4(2)} + 90e^{-.4(1)}$$

c) 3 days

$$90e^{-.4(3)} + 90^{-.4(2)} + 90e^{-4.(1)}$$

d) after ∞ days

just 1 term, S is adding terms

$$a = 90e^{-.4} \qquad r = \frac{a_2}{a_1} = \frac{90e^{-.4(2)}}{90e^{-.4(1)}} = e^{-.4}$$

$$\sum_{n=1}^{\infty} (90e^{-.4})(e^{-.4})^{n-1}$$

e) how many doses left?

$$\lim_{n\to\infty} S_n = \frac{a}{1-r} = \frac{90e^{-.4}}{1-e^{-.4}}$$

## Reoccurring Decimal Places

3. $0.3\overline{12} = 0.3 + 0.012 + 0.00012$

$$= 0.3 + 12\left(\frac{1}{1000}\right) + 12\left(\frac{1}{100000}\right) + \dots$$

$$= 0.3 + 12\left(\frac{1}{100}\right)^2 + 12\left(\frac{1}{100}\right)^3$$

$$= 0.3 + \sum_{n=1}^{\infty} \frac{12}{1000}\left(\frac{1}{100}\right)^{n-1}$$

1 number repeating is $\frac{1}{10}$, 3 is $\frac{1}{1000}$

Written as a series

$$= \frac{3}{10} + \frac{0.012}{1-\frac{1}{100}} \Rightarrow$$ combine to 1 fraction

4. Consecutive terms in geometric series $4c+3$, $4c+2$, $4c+5$. Find $c$.

$$r = \frac{a_{n+1}}{a_n} = \frac{4c+2}{4c+3} = \frac{4c+5}{4c+2}$$ Cross multiply

$$(4c+2)(4c+2) = (4c+5)(4c+3)$$

$\hookrightarrow$ common ratio

$$a_{n+1} \cdot a_n = a_0 r^{n+1} \cdot a_0 \cdot r^n$$

$$a_0 r^{n+1} \cdot a_0 r^n = a_0^2 \cdot r^{n+1+n} = a^2 r^{2n+1} = (ar^n)^2$$

5. $\sum_{n=3}^{\infty} 5\left(\frac{1}{4}\right)^{n-1}$

$S_1 = a_3$

sum 1 term

$S_2 = a_3 + a_4$

$\vdots$

$S_n = a_3 + a_4 + \ldots + a_{N+2}$

$\sum_{n=1}^{\infty} ar^n$

N is specific #

n is an index

$S_N = a_1 + \ldots + a_N$

Partial Sum of Telescopic

6. $\sum_{n=4}^{\infty} \cos\left(\frac{8}{3n}\right) - \cos\left(\frac{8}{3n+3}\right)$

Know it's telescopic bc not geometric ($ar^n$)

a) sum of N terms

$S_N = a_4 + \ldots + a_{N+3}$

$= \left[\cos\left(\frac{8}{12}\right) - \cos\left(\frac{8}{15}\right)\right] + \left[\cos\left(\frac{8}{15}\right) - \cos\left(\frac{8}{18}\right)\right] + \left[\cos\left(\frac{8}{18}\right) - \cos\left(\frac{8}{21}\right)\right]$

cancel    cancel

$+ \ldots + \left[\cos\left(\frac{8}{3(N+2)}\right) - \cos\left(\frac{8}{3(N+2)+3}\right)\right] + \left[\cos\left(\frac{8}{3(N+3)}\right) - \cos\left(\frac{8}{3(N+3)+3}\right)\right]$

cancel

$S_N = \cos\left(\frac{8}{12}\right) - \cos\left(\frac{8}{3(N+3)+3}\right)$

Not always 1st and last term that is left over